WISCONSIN DEPARTMENT OF PUBLIC INSTRUCTION

Tony Evers, PhD, State Superintendent

Joint Hearing
Select Committees on the Common Core State Standards

October 3, 2013 — Room 417 North, State Capitol, Madison

By State Superintendent Tony Evers

Good afternoon. My name is Tony Evers, and I am the State Superintendent of Public Instruction. With me today is Emilie Amundson, director of the Department of Public Instruction's Common Core State Standards Team. I would like to thank the chairs and the committee for the opportunity to speak today on Wisconsin's academic standards.

Just today, I received an email from a teacher in a small Northern Wisconsin school district. She said, "Thank you so much for your strong stand on the Common Core. I see how our math classes have changed for the better — really making students think! Keep up the good work." To me it is really that simple.

Academic standards are a set of expectations for what all students in Wisconsin should know and be able to do in kindergarten through 12th grade. Wisconsin has had academic standards in a variety of subject areas for decades, and we have assessed children in the state on their attainment in reading, math, language arts, science, and social studies since the 1990s.

In the two decades since we have had required standardized testing, the state has set the standards, and students demonstrate their progress toward achieving the state's academic standards through the state's assessments. How schools and districts teach that content — through instruction, curriculum, text book adoption, and more — are all local decisions in our state.

As the independently elected constitutional officer twice entrusted by the people of this state with the authority to supervise public instruction, I adopted the Common Core State Standards in English Language Arts and Mathematics as our Wisconsin state standards in June 2010. I did so because the Common Core State Standards are world class standards that will better prepare all of Wisconsin's children to be ready for college and career.

The Common Core State Standards in English Language Arts and Mathematics:

- challenge students to learn at higher levels in the critical areas of reading, writing, and mathematics;
- are rigorous, clear, and deeper standards;
- are benchmarked to the highest U.S. state and international standards to ensure students are ready to succeed in college and/or careers; and
- are aligned to the expectations of higher education and employers.

To those who are concerned that the Common Core represents too much "federal intrusion" into our state and local decisions, let me say clearly that I was not coerced by the federal government to adopt the Common Core, and I didn't adopt the Common Core in order to qualify for a Race to the Top grant.

On the contrary, Wisconsin's road to adopting higher and better standards began as early as 2007 — long before President Obama and Race to the Top came on the scene. We knew back then — and were consistently told by business leaders, higher education leaders, educators, and even legislators — that the previous standards were not getting the job done.

Our old ELA and math standards were only set at grades four, eight, and 12, leaving districts to fill in the gaps between grades; often a costly process that took tons of staff time and resources. We heard from businesses that students were not prepared with the knowledge and skills to succeed in the workplace and from higher education leaders who felt students were not as prepared as they needed to be to succeed in college. And, we consistently received low marks from national think tanks, like the Fordham Foundation, that compared the rigor of standards across states.

As a result, years before the state-led common core initiative, DPI began the process of revising our state standards in 2007.

- In 2007, Competitive Wisconsin and DPI held a summit for state business leaders, higher education, educators, and legislators, to determine the kinds of knowledge and skills they felt were most important for growing an educated and prepared Wisconsin workforce. As a result of that work, it was clear that Wisconsin's old standards would need to be replaced.
- DPI then partnered with the American Diploma Project and Partnership for 21st Century Skills to convene a task force focused on examining Wisconsin's 1998 Model Academic Standards to assess their college and career readiness. The task force, comprised of educators, legislators, parents, and business representatives, found the 1998 standards outdated for the demands of today. After the work of the Task Force, individual English Language Arts and Mathematics leadership teams began the process of revising the standards in 2008.
- Midway through Wisconsin's standards revision process, the National Governors Association (NGA) and the Council of Chief State School Officers (CCSSO) began a multi-state discussion about the value of partnering across states to create a common set of college and career ready benchmarks for math and English language arts. The argument was similar to the statewide discussion we were already having — what are the fundamental knowledge and skills that all children should know and be able to do in order to succeed in today's global economy? As NGA and CCSSO began their work with governors and state chiefs around the country, they were able to leverage the work already in progress by states like Wisconsin.

In 2009, Wisconsin joined as a partner in this state-led process to create higher, clearer standards that were internationally benchmarked, specific but not prescriptive, and most importantly, would prepare students for college and careers.

Given that we were already well into our own standards revision process, we were ahead of the curve to be able to analyze the Common Core and how it matched up with Wisconsin values. We had teams of educators and stakeholders already engaged in conversations about what Wisconsin students should know and be able to do in these two core subject areas. We were able to use these same teams to examine multiple draft versions of the Common Core against our own Wisconsin blueprint, which previous critics of Wisconsin's standards called "intellectually demanding" and "well aligned" with college and career expectations.

In March of 2010, the DPI held a summit with representatives from higher education, administration, school boards, teachers, and parents focused on providing feedback on the final draft of the Common Core standards. In addition, public comment periods were held, where any stakeholder, from parents, to educators, to community members, had full access to the entire draft Common Core Standards document, and invitations to provide feedback were sent through many channels. Wisconsinites responded, and comments were largely positive and focused more on specifics, many changes of which were incorporated into the final draft, released in late spring of 2010.

All of this work made my decision around the Common Core State Standards a no brainer. The Common Core State Standards were — and still are — the right move for our children and for our state. To date, 45 states, the District of Columbia, and four territories have agreed. In addition to the broad support among K-12 educational organizations, numerous state, national, military, higher education, and parent organizations have offered their support as well, including:

- The U.S. Chamber of Commerce
- The Business Roundtable
- The Partnership for 21st Century Skills
- The Business-Higher Education Forum
- The Association of American Colleges and Universities
- The Military Child Coalition
- The United States Army and
- The National PTA, to name just a few.

Since 2010, hundreds of thousands of students and parents have been working to reach this new, tougher bar, and tens of thousands of educators are using the Common Core State Standards in classrooms across the state. Schools and districts have been working with the new standards to plan curriculum and instruction that is both meaningful for the local community, and for the students they serve. They have been publically available for all citizens for over three years.

Also, you, the Legislature, and the governor have supported the implementation of the Common Core in various ways since 2010.

- While I have the constitutional and statutory authority to adopt standards, in 2011 Wisconsin Act 32, the Legislature and governor explicitly required me to adopt the Common Core Standards. The act directed the Department of Public Instruction to replace the Wisconsin Knowledge and Concepts Examination, which measured the old 1998-era standards, with new pupil assessments that shall "measure mastery of the common core standards."
- 2013 Wisconsin Act 20, this year's biennial budget bill, fully funds the Smarter Balanced Assessments aligned to the Common Core. These new assessments are being piloted across the state this spring, and will be fully implemented next year.
- The Governor's Read to Lead Task Force, which included bipartisan legislative representation, affirmed support for the Common Core State Standards in English Language Arts and considered them to be the underpinning to improving reading outcomes in our state. That report specifically recommended that DPI review and update Wisconsin's Model Early Learning Standards to align with the Common Core.
- 2011 Wisconsin Act 166 established a new state law on educator effectiveness which requires the use of the new, Common Core-aligned assessments for teacher evaluations, and they are a basis of our work with teacher education programs.
- The new state school accountability system, based on the work of the School and District Accountability Design Team that was quad-chaired by the governor, Legislature's education chairs, and me, created report cards for school and districts that will have the new assessments as a key component.
- The Common Core State Standards were a point of discussion during the work of the Governor's Council on College and Workforce Readiness, which included bipartisan legislative representation. The report of the chair of that group, Tim Sullivan, affirmed the importance of the standards, noting their goal of "deeper understanding" of key topics.

The Common Core State Standards are a serious step forward for Wisconsin. They are the foundation of many other transformative efforts to improve student achievement, and our schools. Without strong standards at the foundation,

all of our other efforts — around assessment, accountability, educator effectiveness, early reading — will all fall apart.

Wisconsin's Common Core State Standards are world class, and set a much higher bar for all kids in the state to reach. And we owe it to them to stay the course.

Thank you and I am free to take a few questions.

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Tony Evers is Wisconsin's elected state superintendent of public instruction. An electronic copy of this testimony can be found online at http://commoncore.dpi.wi.gov/files/commoncore/CCSS_common-core-state-standards-testimony.pdf.

COMMON CORE
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Top 5 Reasons to Support the Common Core State Standards

1 Wisconsin's Voice

- Wisconsin educators, parents, community leaders, and the public provided input into the development of the standards before the state adopted them in 2010. Local schools decide how the standards should be taught and which resources to be used in curriculum.

2 Great for Students

- The CCSS are more rigorous than Wisconsin's previous standards and are on par with what is taught in leading countries around the world. They promote creative and critical thinking over rote memorization. They prepare students with the skills they need to succeed in a globally competitive workforce.

3 Great for Schools

- The CCSS are rigorous, clear, and specific at each grade level. Teachers are able to collaborate within and across disciplines to create new, innovative, and more effective ways to actively engage students in learning.

4 Great for Parents

- Because the CCSS are aligned with college and career expectations, students are better prepared for the demands they face after high school. College costs are reduced because there is less need for expensive, remedial, non-credit bearing courses. Students enter the workforce better equipped to earn a livable wage.

5 Great for the Economy

- These higher standards prepare students, our future workforce, with the skills that the workplace demands. A better prepared, more highly skilled workforce attracts new businesses to the state and allows current businesses to hire employees that are ready to work.

To learn more about the CCSS, visit http://standards.dpi.wi.gov/ccss-community/ccss-comunity .

COMMON CORE
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Wisconsin Standards for English Language Arts

From Model Academic Standards (1998) to Common Core State Standards

1 World Class Standards

Common Core State Standards demand more from students. With Wisconsin Model Academic Standards (1998), students were expected to simply "*respond*" while reading. Common Core State Standards expect students to "*analyze*, *delineate*, *integrate*, and *interpret*" while reading.

2 Clearer Standards

Common Core State Standards are clearer than the Wisconsin Model Academic Standards (1998). Wisconsin Model Academic Standards (1998) had only *six* broad standards for what students needed to know and be able to do. This approach often led to confusion about what to teach and when. Common Core State Standards provide over *forty* standards that clearly state what it is students need to know and be able to do at each grade level.

3 More Specific Standards

Common Core State Standards are more specific than the Wisconsin Model Academic Standards (1998). Old Wisconsin Standards had identical standards for fourth graders, eighth graders, and twelfth graders while the Common Core State Standards provide *grade-level standards* that define the exact skills and understandings that students must demonstrate at each grade level.

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1998 Wisconsin Model Academic Standards for ELA

Reading Content Standard

(same standard K-12th grade)

Read and respond to a wide range of writing to build an understanding of written materials, of themselves, and of others

One (of 26 total) READING standards from **Common Core State Standards** for ELA Reading Foundational Skills: Focus on Phonics and Word Recognition (RF 3)	
Grade	*Notice how phonics skills build on one another from kindergarten through 5th grade.
K	**Know and apply grade-level phonics and word analysis skills in decoding words.** • Demonstrate basic knowledge of one-to-one letter-sound correspondences by producing the primary sound or many of the most frequent sounds for each consonant. • Associate the long and short sounds with the common spellings (graphemes) for the five major vowels. • Read common high-frequency words by sight (e.g., *the, of, to, you, she, my, is, are, do, does*). • Distinguish between similarly spelled words by identifying the sounds of the letters that differ.
1	**Know and apply grade-level phonics and word analysis skills in decoding words.** • Know the spelling-sound correspondences for common consonant digraphs. • Decode regularly spelled one-syllable words. • Know final -e and common vowel team conventions for representing long vowel sounds. • Use knowledge that every syllable must have a vowel sound to determine the number of syllables in a printed word. • Decode two-syllable words following basic patterns by breaking the words into syllables. • Read words with inflectional endings. • Recognize and read grade-appropriate irregularly spelled words.
2	**Know and apply grade-level phonics and word analysis skills in decoding words.** • Distinguish long and short vowels when reading regularly spelled one-syllable words. • Know spelling-sound correspondences for additional common vowel teams. • Decode regularly spelled two-syllable words with long vowels. • Decode words with common prefixes and suffixes. • Identify words with inconsistent but common spelling-sound correspondences. • Recognize and read grade-appropriate irregularly spelled words.
3	**Know and apply grade-level phonics and word analysis skills in decoding words.** • Identify and know the meaning of the most common prefixes and derivational suffixes. • Decode words with common Latin suffixes. • Decode multi-syllable words. • Read grade-appropriate irregularly spelled words.
4	**Know and apply grade-level phonics and word analysis skills in decoding words.** • Use combined knowledge of all letter-sound correspondences, syllabication patterns, and morphology (e.g., roots and affixes) to read accurately unfamiliar multisyllabic words in context and out of context.
5	**Know and apply grade-level phonics and word analysis skills in decoding words.** • Use combined knowledge of all letter-sound correspondences, syllabication patterns, and morphology (e.g., roots and affixes) to read accurately unfamiliar multi-syllabic words in context and out of context.

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Wisconsin Standards for Mathematics

From Model Academic Standards (1998) to Common Core State Standards

1 World Class Standards

Learning mathematics with understanding is a focus of the Common Core State Standards. Many of the Common Core State Standards begin with the verb "understand." Students understand a mathematical concept if they can use mathematical reasoning along with a variety of representations, explain the concept to someone else, and apply the concept to another situation. The old Wisconsin standards have less emphasis on these higher order skills.

2 Clearer Standards

Common Core State Standards are clearer than the Wisconsin Model Academic Standards (1998) and provide a specific staircase of skills that build upon one another. Wisconsin Model Academic Standards (1998) were written only for fourth, eighth and twelfth grades. This made it difficult to know what students should master at specific grade levels. For example, the old Wisconsin standards stated that students should be able to "use numbers effectively for various purposes." The student's development of mathematical operations and number sense is never specified.

3 More Specific Standards

Common Core State Standards provide grade-level priorities not found in the Wisconsin Model Academic Standards (1998). Wisconsin Model Academic Standards (1998) have only six total standards and rely instead on a handful of performance indicators that are broad, optional, and appear only at three grade levels (4, 8, and 12).

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Wisconsin Model Academic Standards for Mathemtics (1998)

Number Operations & Relationships Content Standard

(same standard K-12th grade)

Use numbers effectively for various purposes, such as counting, measuring, estimating, and problem solving.

One (of over 20) Common Core State Standard for Mathematics: Operations and Algebraic Thinking to Algebra

Grade	*Notice how mathematics skills build on one another to college and career readiness.
K	Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
1	Understand and apply properties of operations and the relationship between addition and subtraction.
2	Represent and solve problems involving addition and subtraction.
3	Understand properties of multiplication and the relationship between multiplication and division.
4	Use the four operations with whole numbers to solve problems.
5	Write and interpret numerical expressions.
6	Apply and extend previous understandings of arithmetic to algebraic expressions.
7	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
8	Analyze and solve linear equations and pairs of simultaneous linear equations.
HS	Understand solving equations as a process of reasoning and explain the reasoning.

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COMMON CORE STATE STANDARDS WISCONSIN

Wisconsin and the Common Core State Standards for English Language Arts and Mathematics:

What are the Common Core State Standards?

Since 2010, 45 states and the District of Columbia have adopted the Common Core State Standards (CCSS), a set of consistent English language arts and mathematics expectations for what students should know and be able to do in kindergarten through 12th grade. Each state made its local decision to adopt after opportunities to review drafts and voice feedback. Wisconsin was well poised to adopt the CCSS in June of 2010, leveraging the involvement of statewide English language arts and mathematics educator leadership teams in the review process, and gaining statewide support through multiple stakeholder events held in the spring of 2010.

The CCSS are rigorous, internationally-benchmarked English language arts and mathematics standards that are designed to ensure that students leave school with the knowledge and skills needed to succeed in college and careers. The CCSS are NOT a national or state curriculum nor are they federally mandated. They were developed by a team of experts, educators and stakeholders in a process led by the National Governors Association and the Council of Chief State School Officers. Since 2010, the CCSS have garnered support from business, professional, advocacy, and policy stakeholders, and also have the support of national parent organizations.

How will Wisconsin assess the Common Core State Standards?

States will select an aligned assessment to the CCSS where scores will be available sooner than ever before, providing parents and teachers the ability to intervene quickly and support struggling students. Wisconsin's selected state assessment must match the academic demands of the CCSS, reflect the instructional shifts evident in the CCSS, and measure college and career readiness. To that end, Wisconsin is committed to an assessment system aligned to the CCSS in both letter and spirit, which will also include a college entrance assessment such as the ACT. Wisconsin will continue to work with Smarter Balanced Assessment Consortium to develop a system of multiple assessments at grades 3-11, including formative, interim, and summative tools, as well as resources that will assist with further CCSS implementation.

Why share standards across states?

In 2010, the National Governors Association (NGA) and the Council of State School Officers (CCSSO) recognized the opportunity to clearly define the knowledge and skills that will prepare students for the 21st century workplace and will ensure students are truly college and career ready. Leveraging the expertise and experience of educators across 48 states was crucial to developing strong standards that

meet the needs of all students. Working with state partners on the implementation of the CCSS also helps us to learn from one another, share free and available resources, and support students as they move from state to state.

The CCSS provide this guidance and support, while still maintaining the local control so important to districts throughout the state. Since 2010, Wisconsin has begun to link this system of CCSS, instruction and assessment to school accountability measures, educator effectiveness, and accountability for pre-service educator preparation programs. The CCSS provide the foundation for all of these important systems statewide. Additionally, the CCSS provide a framework to Wisconsin educators for research-based instructional practices that will truly transform student learning. This is a significant opportunity for our state.

Example English language arts and mathematics standards:

Academic standards are expectations for what students should know and be able to do in kindergarten through 12th grade. The Common Core State Standards articulate this knowledge and skills in the areas of English language arts and mathematics.

Example English language arts CCSS from grade 3: Determine the main idea of a text; recount the key details and explain how they support the main idea (Informational Reading, 3.2)

Example mathematics CCSS from grade 6: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. (Number System, 6.3)

Where can I find the CCSS?

There are two places to view and download the CCSS.

Council of Chief State School Officers and National Governors Association sponsored website: www.corestandards.org

Wisconsin Department of Public Instruction Common Core State Standards website:

http://commoncore.dpi.wi.gov/

For up to date Wisconsin CCSS news, resources and opportunities, follow us on Twitter @WisDPICCSS

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COMMON CORE STATE STANDARDS WISCONSIN

Common Core State Standards Myths : General

NEW

Myth: The CCSS are not challenging enough.

Fact: The CCSS have dramatically improved the rigor of Wisconsin standards. A 2008 task force of educators, parents, higher education faculty, and business leaders were charged by former-Superintendent Burmaster to examine existing Wisconsin math and English standards and to make recommendations for improving them. This process was facilitated by the American Diploma Project, a non-profit organization responsible for assisting over 30 states examine their state standards for evidence of college and career readiness. At the end of this process in 2008, the task force recommended a complete revision of standards to ensure college and career readiness for all Wisconsin students and submitted a number of key features that should be required of any Wisconsin college and career ready standards. The CCSS meet all of them; a conclusion verified through our own state consideration of the drafts of the CCSS, which included stakeholder meetings and a period of open comment on the draft standards. The Thomas B. Fordham Foundation, a national conservative-leaning education think tank, estimates that Wisconsin experienced one of the highest jumps in rigor from old to new standards when we adopted the CCSS.

Myth: The CCSS only include skills and do not address the importance of content knowledge.

Fact: The CCSS recognize that both content and skills are important.

In English language arts, the CCSS require certain critical content for all students, including: classic fables and stories from around the world, America's Founding Documents, American literature, and Shakespeare. Appropriately, the crucial decisions about what content should be taught are left to local determination.

In mathematics, the CCSS lay a solid foundation in the four operations of addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals. Taken together, these elements support a student's ability to learn and apply more demanding mathematics concepts and procedures. The middle school and high school CCSS call on students to practice applying mathematical ways of thinking to real world issues and challenges; they prepare students to think and reason mathematically. The CCSS set a rigorous definition of college and career readiness, not by piling topic upon topic, but by demanding that students develop a depth of understanding and ability to apply mathematics to new situations, as college students and employees regularly do.

In addition to English language arts and mathematics, the CCSS require that students systematically acquire knowledge in other disciplines through more reading, writing, speaking, and listening than ever before. To assist with this expectation, Wisconsin's adoption of the CCSS includes the CCSS for Literacy in All Subject Areas. The literacy standards assist content area teachers in teaching deeper and richer content knowledge in all subjects. These standards acknowledge the important role of reading, writing, speaking, listening, and language in developing and communicating content knowledge in ways that deepen understanding. For example, the way a student writes a science lab report is very different than the writing necessary to craft a historical essay with support, or a business letter to a prospective employer. Students need to learn the reading, writing, speaking, and listening skills most relevant to

communicating and understanding in each content area, which will lead to better mastery of content across the curriculum.

Myth: The CCSS are not internationally benchmarked.

Fact: International benchmarking played a significant role in the development of the CCSS. In fact, the CCSS include an appendix listing the evidence that was consulted in drafting the CCSS and the international data consulted in the benchmarking process is included in the appendix.

Myth: Other states have repealed the CCSS.

NEW

Fact: At the date of publication (Aug 6, 2013) none of the 45 states who have fully adopted the CCSS have repealed or plan to repeal or replace the CCSS. Though several states are participating in a review of the CCSS, no state has taken the step to "undo" the CCSS.

Myths About Implementation

Myth: The cost of implementing the CCSS represents a huge burden for Wisconsin citizens.

NEW

Fact: The cost of implementing the CCSS is no different than the cost of implementing any set of standards. DPI has not requested any additional funding for CCSS implementation in either the 2011-13 or 2013-15 state budgets. The costs of CCSS-aligned assessments are in line with previous state WKCE assessments required by federal NCLB legislation, and this cost was supported throughout the 2013-15 state budget process. Locally, each Wisconsin district selects instructional materials aligned to standards, and this process is a constantly ongoing cycle. The adoption of the CCSS has been worked into this curriculum adoption cycle locally, just like any other state standards adoption in the past.

Myth: The CCSS tell teachers what to teach.

Fact: The best understanding of what works in the classroom comes from the teachers who are in the classroom. That's why these standards establish *what* students need to learn, but they do not dictate *how* teachers should teach. Instead, individual states, local schools and teachers decide how best to help students reach the CCSS.

Myth: The CCSS amount to a national curriculum for our schools.

Fact: The CCSS are not a curriculum. They are a clear set of expectations for what knowledge and skills will help our students succeed. Local teachers, principals, superintendents and others decide *how* the CCSS are to be met. Teachers continue to devise lesson plans and tailor instruction to the individual needs of the students in their classrooms.

NEW

Myth: The CCSS eliminate competition among textbook publishers and curriculum developers, forcing private and home-school children to use CCSS curriculum.

Fact: In Wisconsin, all public and private schools as well as home-schooled students have complete freedom to select the high quality instructional materials that best meet the needs of their students. The CCSS are simply a set of standards for what students should know and be able to do. There are countless textbook series and curriculum approaches dedicated to meeting these standards, as well as other sets of standards used in the US. This approach is unchanged from past years.

Myth: The federal government will take over ownership of the Common Core State Standards Initiative.

Fact: The federal government does not govern the Common Core State Standards Initiative. The Initiative was and remains a state-led effort supported through the National Governors Association Council of Chief State School Officers. In Wisconsin, all decisions about curriculum and content are made at the local level.

Myth: Implementation of the CCSS will lead to government collection and sharing of private personal and family information via an electronic database.

Fact: Student data are highly protected and our efforts around student data systems fully comply with the law by adopting data privacy and security protections that meet the highest industry standards, exceed federal (Family Educational Rights and Privacy Act or FERPA) requirements, and are designed to ensure that student data are used only for agreed-upon education purposes and not further disclosed. In no case would data never be sold to anyone - ever. The data are owned by the districts, and the DPI will protect it according to federal and state laws. No new data collection is required as part of the Common Core State Standards initiative.

Myths About Process

Myth: The CCSS has not been adequately field tested.

NEW

Fact: Standards are not field-tested; rather they are revised at appropriate intervals. There are countless materials designed to teach standards through strategies, curriculum, textbooks etc, and any attempt to "field test" a set of standards would not test the standards as much as the resources used to teach the standards. The CCSS adoption and revision approach is no different than approaches taken by the majority of states (including Wisconsin under Governor Thompson) in designing and adopting standards. As the CCSS are fully implemented and we begin to analyze the college and career readiness of students in Wisconsin, we will continue to make necessary adjustments and revisions to the standards. Assessments are often field-tested, and currently, Wisconsin is participating in field-testing and piloting the SMARTER assessment aligned to the CCSS.

Myth: No teachers were involved in writing the CCSS.

Fact: The standards drafting process relied on teachers and standards experts from across the country. Wisconsin teachers played an important role in the reviewing and release of the standards.

Myth: The CCSS are not research or evidence based.

Fact: The CCSS have made careful use of a large and growing body of evidence. The evidence base includes scholarly research; surveys on what skills are required of students entering college and workforce training programs; assessment data identifying college-and career-ready performance; and comparisons to standards from high-performing states and nations.

In English language arts, the CCSS build on the firm foundation of the National Assessment of Educational Progress (NAEP) frameworks in Reading and Writing, which draw on extensive scholarly research and evidence. Additionally, the National Reading Panel Report provided guidance to the CCSS, particularly on the Reading Foundations section.

In mathematics, the CCSS draw on recommendations from Trends in International Mathematics and Science Study (TIMSS) and other studies of high-performing countries that the traditional mathematics curriculum must become substantially more coherent and focused to improve student achievement, addressing the problem of a curriculum that is "a mile wide and an inch deep."

Myths About Content and Quality: ELA/Literacy

Myth: The CCSS in English Language Arts are just vague descriptions of skills; they don't include a reading list or any other similar reference to content.

Fact: The CCSS provide specific descriptions and qualities of the sufficiently complex texts educators should be using in each grade level. The CCSS provide guidance for teachers, schools, and districts in making decisions about what texts to use. To meet this goal, the CCSS include excerpted sample texts (Appendix B) that demonstrate the level of text complexity appropriate for grade bands. Teachers, schools, and districts understand the learning demands set out in the CCSS, examine the exemplars for text complexity, and make decisions about what texts will best meet the needs of students. Wisconsin has developed high quality materials and processes educators can use to help select complex texts in line with the text complexity outlined in Appendix B. These resources can be found at http://CCSS.dpi.wi.gov/stn_ela-tchingandlrng

Myth: The CCSS do not emphasize fiction/literature. The standards ask English teachers to teach other subject-area texts.

Fact: The CCSS require certain critical content for all students, including: classic fables and stories from around the world, America's Founding Documents, American literature, and Shakespeare. Appropriately, the remaining crucial decisions about what content should be taught are left to local determination. All reading lists and other content decisions are made at the local level in Wisconsin. In

addition to content coverage, the CCSS require that students systematically acquire knowledge in literature and other disciplines through reading, writing, speaking, and listening.

Much of the reading done in high school, college and the workforce is informational text. To meet this demand, the CCSS call for students to read increasing amounts of informational text as they move toward college and career readiness. This increased amount of nonfiction reading happens through the texts read in all subject areas, including science, social studies, history, and technical subjects to gain critical content knowledge. Fiction/literature remains a critical part of what is studied in English classrooms. Wisconsin has become a national leader in efforts to ensure all subject areas include reading, writing, speaking and listening activities that are relevant to that subject. For more information on this effort, called Disciplinary Literacy in Wisconsin, please visit http://standards.dpi.wi.gov/stn_disciplinaryliteracy

Myths About Content and Quality: Mathematics

NEW

Myth: The CCSS for mathematics do not prepare or require students to learn Algebra in the 8th grade, as many states' current standards do.

Fact: When fully implemented, the CCSS prepare students for Algebra 1 in 8th grade by including the prerequisites for this course in grades K-7. Students who master the K-7 material with sufficient understanding will be able to take Algebra 1 in 8th grade. At the same time, grade 8 standards are also included; these include rigorous algebra and will transition students effectively into a full Algebra 1 course.

The K-7 standards build a strong foundation where students are actively engaged in learning and understanding the prerequisite skills for a successful transition to higher levels of mathematics. Students who have completed 7th grade and mastered the content and skills throughout the 7th grade will be well prepared for algebra in grade 8. Together, the middle school CCSS are robust and provide a coherent and rich preparation for high school mathematics. Fifteen presidents of the professional societies that make up the Conference Board of the Mathematical Sciences have recently signed a statement of "strong support" for the Common Core State Standards for Mathematics (Aug 6, 2013). The statement calls the Common Core State Standards "an auspicious advance in mathematics education."

Myth: Key mathematics topics are missing or appear in the wrong grade.

Fact: The mathematical progressions presented in the CCSS are coherent and based on evidence.

Part of the problem with having 50 different sets of state standards is that today, different states cover different topics at different grade levels. Coming to consensus guarantees that from the viewpoint of any given state, topics will move up or down in the grade level sequence. This is unavoidable. What is important to keep in mind is that the progression in the CCSS is mathematically coherent and leads to college and career readiness at an internationally competitive level.

Myth: The CCSS for mathematics do not require students to learn basic math facts.

Fact: The CCSS specify grade levels by which students are expected to be fluent with basic facts. The CCSS set a rigorous definition for a richer acquisition of mathematical content by asking students to apply the mathematics at a deeper level, gain a conceptual understanding of the mathematics while at the same time obtain the speed and fluency of knowing mathematics facts and algorithms (solve for x).

SECTION 3

Common Core State Standards for English Language Arts

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Introduction

The Common Core State Standards for English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects ("the Standards") are the culmination of an extended, broad-based effort to fulfill the charge issued by the states to create the next generation of K–12 standards in order to help ensure that all students are college and career ready in literacy no later than the end of high school.

The present work, led by the Council of Chief State School Officers (CCSSO) and the National Governors Association (NGA), builds on the foundation laid by states in their decades-long work on crafting high-quality education standards. The Standards also draw on the most important international models as well as research and input from numerous sources, including state departments of education, scholars, assessment developers, professional organizations, educators from kindergarten through college, and parents, students, and other members of the public. In their design and content, refined through successive drafts and numerous rounds of feedback, the Standards represent a synthesis of the best elements of standards-related work to date and an important advance over that previous work.

As specified by CCSSO and NGA, the Standards are (1) research and evidence based, (2) aligned with college and work expectations, (3) rigorous, and (4) internationally benchmarked. A particular standard was included in the document only when the best available evidence indicated that its mastery was essential for college and career readiness in a twenty-first-century, globally competitive society. The Standards are intended to be a living work: as new and better evidence emerges, the Standards will be revised accordingly.

The Standards are an extension of a prior initiative led by CCSSO and NGA to develop College and Career Readiness (CCR) standards in reading, writing, speaking, listening, and language as well as in mathematics. The CCR Reading, Writing, and Speaking and Listening Standards, released in draft form in September 2009, serve, in revised form, as the backbone for the present document. Grade-specific K–12 standards in reading, writing, speaking, listening, and language translate the broad (and, for the earliest grades, seemingly distant) aims of the CCR standards into age- and attainment-appropriate terms.

The Standards set requirements not only for English language arts (ELA) but also for literacy in history/social studies, science, and technical subjects. Just as students must learn to read, write, speak, listen, and use language effectively in a variety of content areas, so too must the Standards specify the literacy skills and understandings required for college and career readiness in multiple disciplines. Literacy standards for grade 6 and above are predicated on teachers of ELA, history/social studies, science, and technical subjects using their content area expertise to help students meet the particular challenges of reading, writing, speaking, listening, and language in their respective fields. It is important to note that the 6–12 literacy standards in history/social studies, science, and technical subjects are not meant to replace content standards in those areas but rather to supplement them. States may incorporate these standards into their standards for those subjects or adopt them as content area literacy standards.

As a natural outgrowth of meeting the charge to define college and career readiness, the Standards also lay out a vision of what it means to be a literate person in the twenty-first century. Indeed, the skills and understandings students are expected to demonstrate have wide applicability outside the classroom or workplace. Students who meet the Standards readily undertake the close, attentive reading that is at the heart of understanding and enjoying complex works of literature. They habitually perform the critical reading necessary to pick carefully through the staggering amount of information available today in print and digitally. They actively seek the wide, deep, and thoughtful engagement with high-quality literary and informational texts that builds knowledge, enlarges experience, and broadens worldviews. They reflexively demonstrate the cogent reasoning and use of evidence that is essential to both private deliberation and responsible citizenship in a democratic republic. In short, students who meet the Standards develop the skills in reading, writing, speaking, and listening that are the foundation for any creative and purposeful expression in language.

June 2, 2010

Key Design Considerations

CCR and grade-specific standards

The CCR standards anchor the document and define general, cross-disciplinary literacy expectations that must be met for students to be prepared to enter college and workforce training programs ready to succeed. The K–12 grade-specific standards define end-of-year expectations and a cumulative progression designed to enable students to meet college and career readiness expectations no later than the end of high school. The CCR and high school (grades 9–12) standards work in tandem to define the college and career readiness line—the former providing broad standards, the latter providing additional specificity. Hence, both should be considered when developing college and career readiness assessments.

Students advancing through the grades are expected to meet each year's grade-specific standards, retain or further develop skills and understandings mastered in preceding grades, and work steadily toward meeting the more general expectations described by the CCR standards.

Grade levels for K–8; grade bands for 9–10 and 11–12

The Standards use individual grade levels in kindergarten through grade 8 to provide useful specificity; the Standards use two-year bands in grades 9–12 to allow schools, districts, and states flexibility in high school course design.

A focus on results rather than means

By emphasizing required achievements, the Standards leave room for teachers, curriculum developers, and states to determine how those goals should be reached and what additional topics should be addressed. Thus, the Standards do not mandate such things as a particular writing process or the full range of metacognitive strategies that students may need to monitor and direct their thinking and learning. Teachers are thus free to provide students with whatever tools and knowledge their professional judgment and experience identify as most helpful for meeting the goals set out in the Standards.

An integrated model of literacy

Although the Standards are divided into Reading, Writing, Speaking and Listening, and Language strands for conceptual clarity, the processes of communication are closely connected, as reflected throughout this document. For example, Writing standard 9 requires that students be able to write about what they read. Likewise, Speaking and Listening standard 4 sets the expectation that students will share findings from their research.

Research and media skills blended into the Standards as a whole

To be ready for college, workforce training, and life in a technological society, students need the ability to gather, comprehend, evaluate, synthesize, and report on information and ideas, to conduct original research in order to answer questions or solve problems, and to analyze and create a high volume and extensive range of print and nonprint texts in media forms old and new. The need to conduct research and to produce and consume media is embedded into every aspect of today's curriculum. In like fashion, research and media skills and understandings are embedded throughout the Standards rather than treated in a separate section.

Shared responsibility for students' literacy development

The Standards insist that instruction in reading, writing, speaking, listening, and language be a shared responsibility within the school. The K–5 standards include expectations for reading, writing, speaking, listening, and language applicable to a range of subjects, including but not limited to ELA. The grades 6–12 standards are divided into two sections, one for ELA and the other for history/social studies, science, and technical subjects. This division reflects the unique, time-honored place of ELA teachers in developing students' literacy skills while at the same time recognizing that teachers in other areas must have a role in this development as well.

Part of the motivation behind the interdisciplinary approach to literacy promulgated by the Standards is extensive research establishing the need for college and career ready students to be proficient in reading complex informational text independently in a variety of content areas. Most of the required reading in college and workforce training programs is informational in structure and challenging in content; postsecondary education programs typically provide students with both a higher volume of such reading than is generally required in K–12 schools and comparatively little scaffolding.

The Standards are not alone in calling for a special emphasis on informational text. The 2009 reading framework of the National Assessment of Educational Progress (NAEP) requires a high and increasing proportion of informational text on its assessment as students advance through the grades.

Distribution of Literary and Informational Passages by Grade in the 2009 NAEP Reading Framework

Grade	Literary	Informational
4	50%	50%
8	45%	55%
12	30%	70%

Source: National Assessment Governing Board. (2008). *Reading framework for the 2009 National Assessment of Educational Progress.* Washington, DC: U.S. Government Printing Office.

The Standards aim to align instruction with this framework so that many more students than at present can meet the requirements of college and career readiness. In K–5, the Standards follow NAEP's lead in balancing the reading of literature with the reading of informational texts, including texts in history/social studies, science, and technical subjects. In accord with NAEP's growing emphasis on informational texts in the higher grades, the Standards demand that a significant amount of reading of informational texts take place in and outside the ELA classroom. Fulfilling the Standards for 6–12 ELA requires much greater attention to a specific category of informational text—literary nonfiction—than has been traditional. Because the ELA classroom must focus on literature (stories, drama, and poetry) as well as literary nonfiction, a great deal of informational reading in grades 6–12 must take place in other classes if the NAEP assessment framework is to be matched instructionally.[1] To measure students' growth toward college and career readiness, assessments aligned with the Standards should adhere to the distribution of texts across grades cited in the NAEP framework.

NAEP likewise outlines a distribution across the grades of the core purposes and types of student writing. The 2011 NAEP framework, like the Standards, cultivates the development of three mutually reinforcing writing capacities: writing to persuade, to explain, and to convey real or imagined experience. Evidence concerning the demands of college and career readiness gathered during development of the Standards concurs with NAEP's shifting emphases: standards for grades 9–12 describe writing in all three forms, but, consistent with NAEP, the overwhelming focus of writing throughout high school should be on arguments and informative/explanatory texts.[2]

Distribution of Communicative Purposes by Grade in the 2011 NAEP Writing Framework

Grade	To Persuade	To Explain	To Convey Experience
4	30%	35%	35%
8	35%	35%	30%
12	40%	40%	20%

Source: National Assessment Governing Board. (2007). *Writing framework for the 2011 National Assessment of Educational Progress, pre-publication edition.* Iowa City, IA: ACT, Inc.

It follows that writing assessments aligned with the Standards should adhere to the distribution of writing purposes across grades outlined by NAEP.

Focus and coherence in instruction and assessment

While the Standards delineate specific expectations in reading, writing, speaking, listening, and language, each standard need not be a separate focus for instruction and assessment. Often, several standards can be addressed by a single rich task. For example, when editing writing, students address Writing standard 5 ("Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach") as well as Language standards 1–3 (which deal with conventions of standard English and knowledge of language). When drawing evidence from literary and informational texts per Writing standard 9, students are also demonstrating their comprehension skill in relation to specific standards in Reading. When discussing something they have read or written, students are also demonstrating their speaking and listening skills. The CCR anchor standards themselves provide another source of focus and coherence.

The same ten CCR anchor standards for Reading apply to both literary and informational texts, including texts in history/social studies, science, and technical subjects. The ten CCR anchor standards for Writing cover numerous text types and subject areas. This means that students can develop mutually reinforcing skills and exhibit mastery of standards for reading and writing across a range of texts and classrooms.

[1]The percentages on the table reflect the sum of student reading, not just reading in ELA settings. Teachers of senior English classes, for example, are not required to devote 70 percent of reading to informational texts. Rather, 70 percent of student reading across the grade should be informational.
[2]As with reading, the percentages in the table reflect the sum of student writing, not just writing in ELA settings.

What is Not Covered by the Standards

The Standards should be recognized for what they are not as well as what they are. The most important intentional design limitations are as follows:

1. The Standards define what all students are expected to know and be able to do, not how teachers should teach. For instance, the use of play with young children is not specified by the Standards, but it is welcome as a valuable activity in its own right and as a way to help students meet the expectations in this document. Furthermore, while the Standards make references to some particular forms of content, including mythology, foundational U.S. documents, and Shakespeare, they do not—indeed, cannot—enumerate all or even most of the content that students should learn. The Standards must therefore be complemented by a well-developed, content-rich curriculum consistent with the expectations laid out in this document.

2. While the Standards focus on what is most essential, they do not describe all that can or should be taught. A great deal is left to the discretion of teachers and curriculum developers. The aim of the Standards is to articulate the fundamentals, not to set out an exhaustive list or a set of restrictions that limits what can be taught beyond what is specified herein.

3. The Standards do not define the nature of advanced work for students who meet the Standards prior to the end of high school. For those students, advanced work in such areas as literature, composition, language, and journalism should be available. This work should provide the next logical step up from the college and career readiness baseline established here.

4. The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

5. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-high school lives.

   Each grade will include students who are still acquiring English. For those students, it is possible to meet the standards in reading, writing, speaking, and listening without displaying native-like control of conventions and vocabulary.

   The Standards should also be read as allowing for the widest possible range of students to participate fully from the outset and as permitting appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities *reading* should allow for the use of Braille, screen-reader technology, or other assistive devices, while *writing* should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, *speaking* and *listening* should be interpreted broadly to include sign language.

6. While the ELA and content area literacy components described herein are critical to college and career readiness, they do not define the whole of such readiness. Students require a wide-ranging, rigorous academic preparation and, particularly in the early grades, attention to such matters as social, emotional, and physical development and approaches to learning. Similarly, the Standards define literacy expectations in history/social studies, science, and technical subjects, but literacy standards in other areas, such as mathematics and health education, modeled on those in this document are strongly encouraged to facilitate a comprehensive, schoolwide literacy program.

Students Who are College and Career Ready in Reading, Writing, Speaking, Listening, and Language

The descriptions that follow are not standards themselves but instead offer a portrait of students who meet the standards set out in this document. As students advance through the grades and master the standards in reading, writing, speaking, listening, and language, they are able to exhibit with increasing fullness and regularity these capacities of the literate individual.

They demonstrate independence.

Students can, without significant scaffolding, comprehend and evaluate complex texts across a range of types and disciplines, and they can construct effective arguments and convey intricate or multifaceted information. Likewise, students are able independently to discern a speaker's key points, request clarification, and ask relevant questions. They build on others' ideas, articulate their own ideas, and confirm they have been understood. Without prompting, they demonstrate command of standard English and acquire and use a wide-ranging vocabulary. More broadly, they become self-directed learners, effectively seeking out and using resources to assist them, including teachers, peers, and print and digital reference materials.

They build strong content knowledge.

Students establish a base of knowledge across a wide range of subject matter by engaging with works of quality and substance. They become proficient in new areas through research and study. They read purposefully and listen attentively to gain both general knowledge and discipline-specific expertise. They refine and share their knowledge through writing and speaking.

They respond to the varying demands of audience, task, purpose, and discipline.

Students adapt their communication in relation to audience, task, purpose, and discipline. They set and adjust purpose for reading, writing, speaking, listening, and language use as warranted by the task. They appreciate nuances, such as how the composition of an audience should affect tone when speaking and how the connotations of words affect meaning. They also know that different disciplines call for different types of evidence (e.g., documentary evidence in history, experimental evidence in science).

They comprehend as well as critique.

Students are engaged and open-minded—but discerning—readers and listeners. They work diligently to understand precisely what an author or speaker is saying, but they also question an author's or speaker's assumptions and premises and assess the veracity of claims and the soundness of reasoning.

They value evidence.

Students cite specific evidence when offering an oral or written interpretation of a text. They use relevant evidence when supporting their own points in writing and speaking, making their reasoning clear to the reader or listener, and they constructively evaluate others' use of evidence.

They use technology and digital media strategically and capably.

Students employ technology thoughtfully to enhance their reading, writing, speaking, listening, and language use. They tailor their searches online to acquire useful information efficiently, and they integrate what they learn using technology with what they learn offline. They are familiar with the strengths and limitations of various technological tools and mediums and can select and use those best suited to their communication goals.

They come to understand other perspectives and cultures.

Students appreciate that the twenty-first-century classroom and workplace are settings in which people from often widely divergent cultures and who represent diverse experiences and perspectives must learn and work together. Students actively seek to understand other perspectives and cultures through reading and listening, and they are able to communicate effectively with people of varied backgrounds. They evaluate other points of view critically and constructively. Through reading great classic and contemporary works of literature representative of a variety of periods, cultures, and worldviews, students can vicariously inhabit worlds and have experiences much different than their own.

How to Read This Document

Overall Document Organization

The Standards comprise three main sections: a comprehensive K–5 section and two content area-specific sections for grades 6–12, one for ELA and one for history/social studies, science, and technical subjects. Three appendices accompany the main document.

Each section is divided into strands. K–5 and 6–12 ELA have Reading, Writing, Speaking and Listening, and Language strands; the 6–12 history/ social studies, science, and technical subjects section focuses on Reading and Writing. Each strand is headed by a strand-specific set of College and Career Readiness Anchor Standards that is identical across all grades and content areas.

Standards for each grade within K–8 and for grades 9–10 and 11–12 follow the CCR anchor standards in each strand. Each grade-specific standard (as these standards are collectively referred to) corresponds to the same-numbered CCR anchor standard. Put another way, each CCR anchor standard has an accompanying grade-specific standard translating the broader CCR statement into grade-appropriate end-of-year expectations.

Individual CCR anchor standards can be identified by their strand, CCR status, and number (R.CCR.6, for example). Individual grade-specific standards can be identified by their strand, grade, and number (or number and letter, where applicable), so that RI.4.3, for example, stands for Reading, Informational Text, grade 4, standard 3 and W.5.1a stands for Writing, grade 5, standard 1a. Strand designations can be found in brackets alongside the full strand title.

Who is responsible for which portion of the Standards

A single K–5 section lists standards for reading, writing, speaking, listening, and language across the curriculum, reflecting the fact that most or all of the instruction students in these grades receive comes from one teacher. Grades 6–12 are covered in two content area-specific sections, the first for the English language arts teacher and the second for teachers of history/social studies, science, and technical subjects. Each section uses the same CCR anchor standards but also includes grade-specific standards tuned to the literacy requirements of the particular discipline(s).

Key Features of the Standards

Reading: Text complexity and the growth of comprehension

The Reading standards place equal emphasis on the sophistication of what students read and the skill with which they read. Standard 10 defines a grade-by-grade "staircase" of increasing text complexity that rises from beginning reading to the college and career readiness level. Whatever they are reading, students must also show a steadily growing ability to discern more from and make fuller use of text, including making an increasing number of connections among ideas and between texts, considering a wider range of textual evidence, and becoming more sensitive to inconsistencies, ambiguities, and poor reasoning in texts.

Writing: Text types, responding to reading, and research

The Standards acknowledge the fact that whereas some writing skills, such as the ability to plan, revise, edit, and publish, are applicable to many types of writing, other skills are more properly defined in terms of specific writing types: arguments, informative/explanatory texts, and narratives. Standard 9 stresses the importance of the writing-reading connection by requiring students to draw upon and write about evidence from literary and informational texts. Because of the centrality of writing to most forms of inquiry, research standards are prominently included in this strand, though skills important to research are infused throughout the document.

Speaking and Listening: Flexible communication and collaboration

Including but not limited to skills necessary for formal presentations, the Speaking and Listening standards require students to develop a range of broadly useful oral communication and interpersonal skills. Students must learn to work together, express and listen carefully to ideas, integrate information from oral, visual, quantitative, and media sources, evaluate what they hear, use media and visual displays strategically to help achieve communicative purposes, and adapt speech to context and task.

Language: Conventions, effective use, and vocabulary

The Language standards include the essential "rules" of standard written and spoken English, but they also approach language as a matter of craft and informed choice among alternatives. The vocabulary standards focus on understanding words and phrases, their relationships, and their nuances and on acquiring new vocabulary, particularly general academic and domain-specific words and phrases.

Appendices A, B, and C

Appendix A contains supplementary material on reading, writing, speaking and listening, and language as well as a glossary of key terms. Appendix B consists of text exemplars illustrating the complexity, quality, and range of reading appropriate for various grade levels with accompanying sample performance tasks. Appendix C includes annotated samples demonstrating at least adequate performance in student writing at various grade levels.

STANDARDS FOR

English Language Arts & Literacy in History/Social Studies, Science, and Technical Subjects

K–5

College and Career Readiness Anchor Standards for Reading

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, and ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse media and formats, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see "Research to Build and Present Knowledge" in Writing and "Comprehension and Collaboration" in Speaking and Listening for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

To build a foundation for college and career readiness, students must read widely and deeply from among a broad range of high-quality, increasingly challenging literary and informational texts. Through extensive reading of stories, dramas, poems, and myths from diverse cultures and different time periods, students gain literary and cultural knowledge as well as familiarity with various text structures and elements. By reading texts in history/social studies, science, and other disciplines, students build a foundation of knowledge in these fields that will also give them the background to be better readers in all content areas. Students can only gain this foundation when the curriculum is intentionally and coherently structured to develop rich content knowledge within and across grades. Students also acquire the habits of reading independently and closely, which are essential to their future success.

Reading Standards for Literature K–5

RL

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details		
1. With prompting and support, ask and answer questions about key details in a text.	1. Ask and answer questions about key details in a text.	1. Ask and answer such questions as *who, what, where, when, why,* and *how* to demonstrate understanding of key details in a text.
2. With prompting and support, retell familiar stories, including key details.	2. Retell stories, including key details, and demonstrate understanding of their central message or lesson.	2. Recount stories, including fables and folktales from diverse cultures, and determine their central message, lesson, or moral.
3. With prompting and support, identify characters, settings, and major events in a story.	3. Describe characters, settings, and major events in a story, using key details.	3. Describe how characters in a story respond to major events and challenges.
Craft and Structure		
4. Ask and answer questions about unknown words in a text.	4. Identify words and phrases in stories or poems that suggest feelings or appeal to the senses.	4. Describe how words and phrases (e.g., regular beats, alliteration, rhymes, repeated lines) supply rhythm and meaning in a story, poem, or song.
5. Recognize common types of texts (e.g., storybooks, poems).	5. Explain major differences between books that tell stories and books that give information, drawing on a wide reading of a range of text types.	5. Describe the overall structure of a story, including describing how the beginning introduces the story and the ending concludes the action.
6. With prompting and support, name the author and illustrator of a story and define the role of each in telling the story.	6. Identify who is telling the story at various points in a text.	6. Acknowledge differences in the points of view of characters, including by speaking in a different voice for each character when reading dialogue aloud.
Integration of Knowledge and Ideas		
7. With prompting and support, describe the relationship between illustrations and the story in which they appear (e.g., what moment in a story an illustration depicts).	7. Use illustrations and details in a story to describe its characters, setting, or events.	7. Use information gained from the illustrations and words in a print or digital text to demonstrate understanding of its characters, setting, or plot.
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. With prompting and support, compare and contrast the adventures and experiences of characters in familiar stories.	9. Compare and contrast the adventures and experiences of characters in stories.	9. Compare and contrast two or more versions of the same story (e.g., Cinderella stories) by different authors or from different cultures.
Range of Reading and Level of Text Complexity		
10. Actively engage in group reading activities with purpose and understanding.	10. With prompting and support, read prose and poetry of appropriate complexity for grade 1.	10. By the end of the year, read and comprehend literature, including stories and poetry, in the grades 2–3 text complexity band proficiently, with scaffolding as needed at the high end of the range.

Reading Standards for Literature K–5

RL

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details		
1. Ask and answer questions to demonstrate understanding of a text, referring explicitly to the text as the basis for the answers.	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	1. Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.
2. Recount stories, including fables, folktales, and myths from diverse cultures; determine the central message, lesson, or moral and explain how it is conveyed through key details in the text.	2. Determine a theme of a story, drama, or poem from details in the text; summarize the text.	2. Determine a theme of a story, drama, or poem from details in the text, including how characters in a story or drama respond to challenges or how the speaker in a poem reflects upon a topic; summarize the text.
3. Describe characters in a story (e.g., their traits, motivations, or feelings) and explain how their actions contribute to the sequence of events.	3. Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text (e.g., a character's thoughts, words, or actions).	3. Compare and contrast two or more characters, settings, or events in a story or drama, drawing on specific details in the text (e.g., how characters interact).
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, distinguishing literal from nonliteral language.	4. Determine the meaning of words and phrases as they are used in a text, including those that allude to significant characters found in mythology (e.g., Herculean).	4. Determine the meaning of words and phrases as they are used in a text, including figurative language such as metaphors and similes.
5. Refer to parts of stories, dramas, and poems when writing or speaking about a text, using terms such as chapter, scene, and stanza; describe how each successive part builds on earlier sections.	5. Explain major differences between poems, drama, and prose, and refer to the structural elements of poems (e.g., verse, rhythm, meter) and drama (e.g., casts of characters, settings, descriptions, dialogue, stage directions) when writing or speaking about a text.	5. Explain how a series of chapters, scenes, or stanzas fits together to provide the overall structure of a particular story, drama, or poem.
6. Distinguish their own point of view from that of the narrator or those of the characters.	6. Compare and contrast the point of view from which different stories are narrated, including the difference between first- and third-person narrations.	6. Describe how a narrator's or speaker's point of view influences how events are described.
Integration of Knowledge and Ideas		
7. Explain how specific aspects of a text's illustrations contribute to what is conveyed by the words in a story (e.g., create mood, emphasize aspects of a character or setting).	7. Make connections between the text of a story or drama and a visual or oral presentation of the text, identifying where each version reflects specific descriptions and directions in the text.	7. Analyze how visual and multimedia elements contribute to the meaning, tone, or beauty of a text (e.g., graphic novel, multimedia presentation of fiction, folktale, myth, poem).
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast the themes, settings, and plots of stories written by the same author about the same or similar characters (e.g., in books from a series).	9. Compare and contrast the treatment of similar themes and topics (e.g., opposition of good and evil) and patterns of events (e.g., the quest) in stories, myths, and traditional literature from different cultures.	9. Compare and contrast stories in the same genre (e.g., mysteries and adventure stories) on their approaches to similar themes and topics.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, at the high end of the grades 2–3 text complexity band independently and proficiently.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, in the grades 4–5 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poetry, at the high end of the grades 4–5 text complexity band independently and proficiently.

Reading Standards for Informational Text K–5

RI

Kindergartners:	Grade 1 students:	Grade 2 students:
Key Ideas and Details		
1. With prompting and support, ask and answer questions about key details in a text.	1. Ask and answer questions about key details in a text.	1. Ask and answer such questions as *who, what, where, when, why,* and *how* to demonstrate understanding of key details in a text.
2. With prompting and support, identify the main topic and retell key details of a text.	2. Identify the main topic and retell key details of a text.	2. Identify the main topic of a multiparagraph text as well as the focus of specific paragraphs within the text.
3. With prompting and support, describe the connection between two individuals, events, ideas, or pieces of information in a text.	3. Describe the connection between two individuals, events, ideas, or pieces of information in a text.	3. Describe the connection between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text.
Craft and Structure		
4. With prompting and support, ask and answer questions about unknown words in a text.	4. Ask and answer questions to help determine or clarify the meaning of words and phrases in a text.	4. Determine the meaning of words and phrases in a text relevant to a *grade 2 topic or subject area.*
5. Identify the front cover, back cover, and title page of a book.	5. Know and use various text features (e.g., headings, tables of contents, glossaries, electronic menus, icons) to locate key facts or information in a text.	5. Know and use various text features (e.g., captions, bold print, subheadings, glossaries, indexes, electronic menus, icons) to locate key facts or information in a text efficiently.
6. Name the author and illustrator of a text and define the role of each in presenting the ideas or information in a text.	6. Distinguish between information provided by pictures or other illustrations and information provided by the words in a text.	6. Identify the main purpose of a text, including what the author wants to answer, explain, or describe.
Integration of Knowledge and Ideas		
7. With prompting and support, describe the relationship between illustrations and the text in which they appear (e.g., what person, place, thing, or idea in the text an illustration depicts).	7. Use the illustrations and details in a text to describe its key ideas.	7. Explain how specific images (e.g., a diagram showing how a machine works) contribute to and clarify a text.
8. With prompting and support, identify the reasons an author gives to support points in a text.	8. Identify the reasons an author gives to support points in a text.	8. Describe how reasons support specific points the author makes in a text.
9. With prompting and support, identify basic similarities in and differences between two texts on the same topic (e.g., in illustrations, descriptions, or procedures).	9. Identify basic similarities in and differences between two texts on the same topic (e.g., in illustrations, descriptions, or procedures).	9. Compare and contrast the most important points presented by two texts on the same topic.
Range of Reading and Level of Text Complexity		
10. Actively engage in group reading activities with purpose and understanding.	10. With prompting and support, read informational texts appropriately complex for grade 1.	10. By the end of year, read and comprehend informational texts, including history/social studies, science, and technical texts, in the grades 2–3 text complexity band proficiently, with scaffolding as needed at the high end of the range.

Reading Standards for Informational Text K–5

RI

Grade 3 students:	Grade 4 students:	Grade 5 students:
Key Ideas and Details		
1. Ask and answer questions to demonstrate understanding of a text, referring explicitly to the text as the basis for the answers.	1. Refer to details and examples in a text when explaining what the text says explicitly and when drawing inferences from the text.	1. Quote accurately from a text when explaining what the text says explicitly and when drawing inferences from the text.
2. Determine the main idea of a text; recount the key details and explain how they support the main idea.	2. Determine the main idea of a text and explain how it is supported by key details; summarize the text.	2. Determine two or more main ideas of a text and explain how they are supported by key details; summarize the text.
3. Describe the relationship between a series of historical events, scientific ideas or concepts, or steps in technical procedures in a text, using language that pertains to time, sequence, and cause/effect.	3. Explain events, procedures, ideas, or concepts in a historical, scientific, or technical text, including what happened and why, based on specific information in the text.	3. Explain the relationships or interactions between two or more individuals, events, ideas, or concepts in a historical, scientific, or technical text based on specific information in the text.
Craft and Structure		
4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 3 topic or subject area*.	4. Determine the meaning of general academic and domain-specific words or phrases in a text relevant to a *grade 4 topic or subject area*.	4. Determine the meaning of general academic and domain-specific words and phrases in a text relevant to a *grade 5 topic or subject area*.
5. Use text features and search tools (e.g., key words, sidebars, hyperlinks) to locate information relevant to a given topic efficiently.	5. Describe the overall structure (e.g., chronology, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in a text or part of a text.	5. Compare and contrast the overall structure (e.g., chronology, comparison, cause/effect, problem/solution) of events, ideas, concepts, or information in two or more texts.
6. Distinguish their own point of view from that of the author of a text.	6. Compare and contrast a firsthand and secondhand account of the same event or topic; describe the differences in focus and the information provided.	6. Analyze multiple accounts of the same event or topic, noting important similarities and differences in the point of view they represent.
Integration of Knowledge and Ideas		
7. Use information gained from illustrations (e.g., maps, photographs) and the words in a text to demonstrate understanding of the text (e.g., where, when, why, and how key events occur).	7. Interpret information presented visually, orally, or quantitatively (e.g., in charts, graphs, diagrams, time lines, animations, or interactive elements on Web pages) and explain how the information contributes to an understanding of the text in which it appears.	7. Draw on information from multiple print or digital sources, demonstrating the ability to locate an answer to a question quickly or to solve a problem efficiently.
8. Describe the logical connection between particular sentences and paragraphs in a text (e.g., comparison, cause/effect, first/second/third in a sequence).	8. Explain how an author uses reasons and evidence to support particular points in a text.	8. Explain how an author uses reasons and evidence to support particular points in a text, identifying which reasons and evidence support which point(s).
9. Compare and contrast the most important points and key details presented in two texts on the same topic.	9. Integrate information from two texts on the same topic in order to write or speak about the subject knowledgeably.	9. Integrate information from several texts on the same topic in order to write or speak about the subject knowledgeably.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend informational texts, including history/social studies, science, and technical texts, at the high end of the grades 2–3 text complexity band independently and proficiently.	10. By the end of year, read and comprehend informational texts, including history/social studies, science, and technical texts, in the grades 4–5 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend informational texts, including history/social studies, science, and technical texts, at the high end of the grades 4–5 text complexity band independently and proficiently.

Reading Standards: Foundational Skills (K–5)

RF

These standards are directed toward fostering students' understanding and working knowledge of concepts of print, the alphabetic principle, and other basic conventions of the English writing system. These foundational skills are not an end in and of themselves; rather, they are necessary and important components of an effective, comprehensive reading program designed to develop proficient readers with the capacity to comprehend texts across a range of types and disciplines. Instruction should be differentiated: good readers will need much less practice with these concepts than struggling readers will. The point is to teach students what they need to learn and not what they already know—to discern when particular children or activities warrant more or less attention.

Note: In kindergarten, children are expected to demonstrate increasing awareness and competence in the areas that follow.

Kindergartners:	Grade 1 students:
Print Concepts	
1. Demonstrate understanding of the organization and basic features of print. a. Follow words from left to right, top to bottom, and page by page. b. Recognize that spoken words are represented in written language by specific sequences of letters. c. Understand that words are separated by spaces in print. d. Recognize and name all upper- and lowercase letters of the alphabet.	1. Demonstrate understanding of the organization and basic features of print. a. Recognize the distinguishing features of a sentence (e.g., first word, capitalization, ending punctuation).
Phonological Awareness	
2. Demonstrate understanding of spoken words, syllables, and sounds (phonemes). a. Recognize and produce rhyming words. b. Count, pronounce, blend, and segment syllables in spoken words. c. Blend and segment onsets and rimes of single-syllable spoken words. d. Isolate and pronounce the initial, medial vowel, and final sounds (phonemes) in three-phoneme (consonent-vowel-consonent, or CVC) words.* (This does not include CVCs ending with /l/, /r/, or /x/.) e. Add or substitute individual sounds (phonemes) in simple, one-syllable words to make new words.	2. Demonstrate understanding of spoken words, syllables, and sounds (phonemes). a. Distinguish long from short vowel sounds in spoken single-syllable words. b. Orally produce single-syllable words by blending sounds (phonemes), including consonant blends. c. Isolate and pronounce initial, medial vowel, and final sounds (phonemes) in spoken single-syllable words. d. Segment spoken single-syllable words into their complete sequence of individual sounds (phonemes).

*Words, syllables, or phonemes written in /slashes/ refer to their pronunciation or phonology.
Thus, /CVC/ is a word with three phonemes regardless of the number of letters in the spelling of the word.

Reading Standards: Foundational Skills (K–5)

RF

Note: In kindergarten children are expected to demonstrate increasing awareness and competence in the areas that follow.

Kindergartners:	Grade 1 students:	Grade 2 students:
Phonics and Word Recognition		
3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Demonstrate basic knowledge of one-to-one letter-sound correspondences by producing the primary or many of the most frequent sound for each consonant. b. Associate the long and short sounds with common spellings (graphemes) for the five major vowels. c. Read common high-frequency words by sight (e.g., *the, of, to, you, she, my, is, are, do, does*). d. Distinguish between similarly spelled words by identifying the sounds of the letters that differ.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Know the spelling-sound correspondences for common consonant digraphs. b. Decode regularly spelled one-syllable words. c. Know final -e and common vowel team conventions for representing long vowel sounds. d. Use knowledge that every syllable must have a vowel sound to determine the number of syllables in a printed word. e. Decode two-syllable words following basic patterns by breaking the words into syllables. f. Read words with inflectional endings. g. Recognize and read grade-appropriate irregularly spelled words.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Distinguish long and short vowels when reading regularly spelled one-syllable words. b. Know spelling-sound correspondences for additional common vowel teams. c. Decode regularly spelled two-syllable words with long vowels. d. Decode words with common prefixes and suffixes. e. Identify words with inconsistent but common spelling-sound correspondences. f. Recognize and read grade-appropriate irregularly spelled words.
Fluency		
4. Read emergent-reader texts with purpose and understanding.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level text orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level text orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.

Reading Standards: Foundational Skills (K–5)

RF

Grade 3 students:	Grade 4 students:	Grade 5 students:
Phonics and Word Recognition		
3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Identify and know the meaning of the most common prefixes and derivational suffixes. b. Decode words with common Latin suffixes. c. Decode multisyllable words. d. Read grade-appropriate irregularly spelled words.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Use combined knowledge of all letter-sound correspondences, syllabication patterns, and morphology (e.g., roots and affixes) to read accurately unfamiliar multisyllabic words in context and out of context.	3. Know and apply grade-level phonics and word analysis skills in decoding words. a. Use combined knowledge of all letter-sound correspondences, syllabication patterns, and morphology (e.g., roots and affixes) to read accurately unfamiliar multisyllabic words in context and out of context.
Fluency		
4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level prose and poetry orally with accuracy, appropriate rate, and expression on successive readings c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level prose and poetry orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.	4. Read with sufficient accuracy and fluency to support comprehension. a. Read on-level text with purpose and understanding. b. Read on-level prose and poetry orally with accuracy, appropriate rate, and expression on successive readings. c. Use context to confirm or self-correct word recognition and understanding, rereading as necessary.

College and Career Readiness Anchor Standards for Writing

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

Note on range and content of student writing

To build a foundation for college and career readiness, students need to learn to use writing as a way of offering and supporting opinions, demonstrating understanding of the subjects they are studying, and conveying real and imagined experiences and events. They learn to appreciate that a key purpose of writing is to communicate clearly to an external, sometimes unfamiliar audience, and they begin to adapt the form and content of their writing to accomplish a particular task and purpose. They develop the capacity to build knowledge on a subject through research projects and to respond analytically to literary and informational sources. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and extended time frames throughout the year.

Writing Standards K–5

W

The following standards for K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* The expected growth in student writing ability is reflected both in the standards themselves and in the collection of annotated student writing samples in Appendix C.

Kindergartners:	Grade 1 students:	Grade 2 students:
Text Types and Purposes		
1. Use a combination of drawing, dictating, and writing to compose opinion pieces in which they tell a reader the topic or the name of the book they are writing about and state an opinion or preference about the topic or book (e.g., *My favorite book is . . .*).	1. Write opinion pieces in which they introduce the topic or name the book they are writing about, state an opinion, supply a reason for the opinion, and provide some sense of closure.	1. Write opinion pieces in which they introduce the topic or book they are writing about, state an opinion, supply reasons that support the opinion, use linking words (e.g., *because, and, also*) to connect opinion and reasons, and provide a concluding statement or section.
2. Use a combination of drawing, dictating, and writing to compose informative/explanatory texts in which they name what they are writing about and supply some information about the topic.	2. Write informative/explanatory texts in which they name a topic, supply some facts about the topic, and provide some sense of closure.	2. Write informative/explanatory texts in which they introduce a topic, use facts and definitions to develop points, and provide a concluding statement or section.
3. Use a combination of drawing, dictating, and writing to narrate a single event or several loosely linked events, tell about the events in the order in which they occurred, and provide a reaction to what happened.	3. Write narratives in which they recount two or more appropriately sequenced events, include some details regarding what happened, use temporal words to signal event order, and provide some sense of closure.	3. Write narratives in which they recount a well-elaborated event or short sequence of events, include details to describe actions, thoughts, and feelings, use temporal words to signal event order, and provide a sense of closure.
Production and Distribution of Writing		
4. (Begins in grade 3)	4. (Begins in grade 3)	4. (Begins in grade 3)
5. With guidance and support from adults, respond to questions and suggestions from peers and add details to strengthen writing as needed.	5. With guidance and support from adults, focus on a topic, respond to questions and suggestions from peers, and add details to strengthen writing as needed.	5. With guidance and support from adults and peers, focus on a topic and strengthen writing as needed by revising and editing.
6. With guidance and support from adults, explore a variety of digital tools to produce and publish writing, including in collaboration with peers.	6. With guidance and support from adults, use a variety of digital tools to produce and publish writing, including in collaboration with peers.	6. With guidance and support from adults, use a variety of digital tools to produce and publish writing, including in collaboration with peers.
Research to Build and Present Knowledge		
7. Participate in shared research and writing projects (e.g., explore a number of books by a favorite author and express opinions about them).	7. Participate in shared research and writing projects (e.g., explore a number of "how-to" books on a given topic and use them to write a sequence of instructions).	7. Participate in shared research and writing projects (e.g., read a number of books on a single topic to produce a report; record science observations).
8. With guidance and support from adults, recall information from experiences or gather information from provided sources to answer a question.	8. With guidance and support from adults, recall information from experiences or gather information from provided sources to answer a question.	8. Recall information from experiences or gather information from provided sources to answer a question.
9. (Begins in grade 4)	9. (Begins in grade 4)	9. (Begins in grade 4)
Range of Writing		
10. (Begins in grade 3)	10. (Begins in grade 3)	10. (Begins in grade 3)

Writing Standards K–5

W

Grade 3 students:	Grade 4 students:	Grade 5 students:
Text Types and Purposes		
1. Write opinion pieces on topics or texts, supporting a point of view with reasons. a. Introduce the topic or text they are writing about, state an opinion, and create an organizational structure that lists reasons. b. Provide reasons that support the opinion. c. Use linking words and phrases (e.g., *because, therefore, since, for example*) to connect opinion and reasons. d. Provide a concluding statement or section.	1. Write opinion pieces on topics or texts, supporting a point of view with reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which related ideas are grouped to support the writer's purpose. b. Provide reasons that are supported by facts and details. c. Link opinion and reasons using words and phrases (e.g., *for instance, in order to, in addition*). d. Provide a concluding statement or section related to the opinion presented.	1. Write opinion pieces on topics or texts, supporting a point of view with reasons and information. a. Introduce a topic or text clearly, state an opinion, and create an organizational structure in which ideas are logically grouped to support the writer's purpose. b. Provide logically ordered reasons that are supported by facts and details. c. Link opinion and reasons using words, phrases, and clauses (e.g., *consequently, specifically*). d. Provide a concluding statement or section related to the opinion presented.
2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic and group related information together; include illustrations when useful to aiding comprehension. b. Develop the topic with facts, definitions, and details. c. Use linking words and phrases (e.g., *also, another, and, more, but*) to connect ideas within categories of information. d. Provide a concluding statement or section.	2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic clearly and group related information in paragraphs and sections; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. c. Link ideas within categories of information using words and phrases (e.g., *another, for example, also, because*). d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Provide a concluding statement or section related to the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas and information clearly. a. Introduce a topic clearly, provide a general observation and focus, and group related information logically; include formatting (e.g., headings), illustrations, and multimedia when useful to aiding comprehension. b. Develop the topic with facts, definitions, concrete details, quotations, or other information and examples related to the topic. c. Link ideas within and across categories of information using words, phrases, and clauses (e.g., *in contrast, especially*). d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Provide a concluding statement or section related to the information or explanation presented.
3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Establish a situation and introduce a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use dialogue and descriptions of actions, thoughts, and feelings to develop experiences and events or show the response of characters to situations. c. Use temporal words and phrases to signal event order. d. Provide a sense of closure.	3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Orient the reader by establishing a situationand introducing a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use dialogue and description to develop experiences and events or show the responses of characters to situations. c. Use a variety of transitional words and phrases to manage the sequence of events. d. Use concrete words and phrases and sensory details to convey experiences and events precisely. e. Provide a conclusion that follows from the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, descriptive details, and clear event sequences. a. Orient the reader by establishing a situation and introducing a narrator and/or characters; organize an event sequence that unfolds naturally. b. Use narrative techniques, such as dialogue, description, and pacing, to develop experiences and events or show the responses of characters to situations. c. Use a variety of transitional words, phrases, and clauses to manage the sequence of events. d. Use concrete words and phrases and sensory details to convey experiences and events precisely. e. Provide a conclusion that follows from the narrated experiences or events.

Writing Standards K–5

W

Grade 3 students:	Grade 4 students:	Grade 5 students:
Production and Distribution of Writing		
4. With guidance and support from adults, produce writing in which the development and organization are appropriate to task and purpose. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development and organization are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development and organization are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, and editing. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 3 on pages 28 and 29.)	5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, and editing. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 4 on pages 28 and 29.)	5. With guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 5 on pages 28 and 29.)
6. With guidance and support from adults, use technology to produce and publish writing (using keyboarding skills) as well as to interact and collaborate with others.	6. With some guidance and support from adults, use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of one page in a single sitting.	6. With some guidance and support from adults, use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of two pages in a single sitting.
Research to Build and Present Knowledge		
7. Conduct short research projects that build knowledge about a topic.	7. Conduct short research projects that build knowledge through investigation of different aspects of a topic.	7. Conduct short research projects that use several sources to build knowledge through investigation of different aspects of a topic.
8. Recall information from experiences or gather information from print and digital sources; take brief notes on sources and sort evidence into provided categories.	8. Recall relevant information from experiences or gather relevant information from print and digital sources; take notes and categorize information, and provide a list of sources.	8. Recall relevant information from experiences or gather relevant information from print and digital sources; summarize or paraphrase information in notes and finished work, and provide a list of sources.
9. (Begins in grade 4)	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 4 Reading standards* to literature (e.g., "Describe in depth a character, setting, or event in a story or drama, drawing on specific details in the text [e.g., a character's thoughts, words, or actions]."). b. Apply *grade 4 Reading standards* to informational texts (e.g., "Explain how an author uses reasons and evidence to support particular points in a text").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 5 Reading standards* to literature (e.g., "Compare and contrast two or more characters, settings, or events in a story or a drama, drawing on specific details in the text [e.g., how characters interact]"). b. Apply *grade 5 Reading standards* to informational texts (e.g., "Explain how an author uses reasons and evidence to support particular points in a text, identifying which reasons and evidence support which point[s]").
Range of Writing		
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

College and Career Readiness Anchor Standards for Speaking and Listening

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Note on range and content of student speaking and listening

To build a foundation for college and career readiness, students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner. Being productive members of these conversations requires that students contribute accurate, relevant information; respond to and develop what others have said; make comparisons and contrasts; and analyze and synthesize a multitude of ideas in various domains.

New technologies have broadened and expanded the role that speaking and listening play in acquiring and sharing knowledge and have tightened their link to other forms of communication. Digital texts confront students with the potential for continually updated content and dynamically changing combinations of words, graphics, images, hyperlinks, and embedded video and audio.

Speaking and Listening Standards K–5

SL

The following standards for K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Kindergartners:	Grade 1 students:	Grade 2 students:
Comprehension and Collaboration		
1. Participate in collaborative conversations with diverse partners about *kindergarten topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others and taking turns speaking about the topics and texts under discussion). b. Continue a conversation through multiple exchanges.	1. Participate in collaborative conversations with diverse partners about *grade 1 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by responding to the comments of others through multiple exchanges. c. Ask questions to clear up any confusion about the topics and texts under discussion.	1. Participate in collaborative conversations with diverse partners about *grade 2 topics and texts* with peers and adults in small and larger groups. a. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). b. Build on others' talk in conversations by linking their comments to the remarks of others. c. Ask for clarification and further explanation as needed about the topics and texts under discussion.
2. Confirm understanding of a text read aloud or information presented orally or through other media by asking and answering questions about key details and requesting clarification if something is not understood.	2. Ask and answer questions about key details in a text read aloud or information presented orally or through other media.	2. Recount or describe key ideas or details from a text read aloud or information presented orally or through other media.
3. Ask and answer questions in order to seek help, get information, or clarify something that is not understood.	3. Ask and answer questions about what a speaker says in order to gather additional information or clarify something that is not understood.	3. Ask and answer questions about what a speaker says in order to clarify comprehension, gather additional information, or deepen understanding of a topic or issue.
Presentation of Knowledge and Ideas		
4. Describe familiar people, places, things, and events and, with prompting and support, provide additional detail.	4. Describe people, places, things, and events with relevant details, expressing ideas and feelings clearly.	4. Tell a story or recount an experience with appropriate facts and relevant, descriptive details, speaking audibly in coherent sentences.
5. Add drawings or other visual displays to descriptions as desired to provide additional detail.	5. Add drawings or other visual displays to descriptions when appropriate to clarify ideas, thoughts, and feelings.	5. Create audio recordings of stories or poems; add drawings or other visual displays to stories or recounts of experiences when appropriate to clarify ideas, thoughts, and feelings.
6. Speak audibly and express thoughts, feelings, and ideas clearly.	6. Produce complete sentences when appropriate to task and situation. (See grade 1 Language standards 1 and 3 on page 26 for specific expectations.)	6. Produce complete sentences when appropriate to task and situation in order to provide requested detail or clarification. (See grade 2 Language standards 1 and 3 on pages 26 and 27 for specific expectations.)

Speaking and Listening Standards K–5

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Grade 3 students:	Grade 4 students:	Grade 5 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 3 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions (e.g., gaining the floor in respectful ways, listening to others with care, speaking one at a time about the topics and texts under discussion). c. Ask questions to check understanding of information presented, stay on topic, and link their comments to the remarks of others. d. Explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 4 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others. d. Review the key ideas expressed and explain their own ideas and understanding in light of the discussion.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 5 topics and texts*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion. b. Follow agreed-upon rules for discussions and carry out assigned roles. c. Pose and respond to specific questions by making comments that contribute to the discussion and elaborate on the remarks of others. d. Review the key ideas expressed and draw conclusions in light of information and knowledge gained from the discussions.
2. Determine the main ideas and supporting details of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	2. Paraphrase portions of a text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.	2. Summarize a written text read aloud or information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Ask and answer questions about information from a speaker, offering appropriate elaboration and detail.	3. Identify the reasons and evidence a speaker provides to support particular points.	3. Summarize the points a speaker makes and explain how each claim is supported by reasons and evidence.
Presentation of Knowledge and Ideas		
4. Report on a topic or text, tell a story, or recount an experience with appropriate facts and relevant, descriptive details, speaking clearly at an understandable pace.	4. Report on a topic or text, tell a story, or recount an experience in an organized manner, using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.	4. Report on a topic or text or present an opinion, sequencing ideas logically and using appropriate facts and relevant, descriptive details to support main ideas or themes; speak clearly at an understandable pace.
5. Create engaging audio recordings of stories or poems that demonstrate fluid reading at an understandable pace; add visual displays when appropriate to emphasize or enhance certain facts or details.	5. Add audio recordings and visual displays to presentations when appropriate to enhance the development of main ideas or themes.	5. Include multimedia components (e.g., graphics, sound) and visual displays in presentations when appropriate to enhance the development of main ideas or themes.
6. Speak in complete sentences when appropriate to task and situation in order to provide requested detail or clarification. (See grade 3 Language standards 1 and 3 on pages 28 and 29 for specific expectations.)	6. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion); use formal English when appropriate to task and situation. (See grade 4 Language standards 1 on pages 28 and 29 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, using formal English when appropriate to task and situation. (See grade 5 Language standards 1 and 3 on pages 28 and 29 for specific expectations.)

College and Career Readiness Anchor Standards for Language

The K–5 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when encountering an unknown term important to comprehension or expression.

Note on range and content of student language use

To build a foundation for college and career readiness in language, students must gain control over many conventions of standard English grammar, usage, and mechanics as well as learn other ways to use language to convey meaning effectively. They must also be able to determine or clarify the meaning of grade-appropriate words encountered through listening, reading, and media use; come to appreciate that words have nonliteral meanings, shadings of meaning, and relationships to other words; and expand their vocabulary in the course of studying content. The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, effective language use, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Language Standards K–5

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The following standards for grades K–5 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*). See the table on page 30 for a complete list and Appendix A for an example of how these skills develop in sophistication.

Kindergartners:	Grade 1 students:	Grade 2 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Print many upper- and lowercase letters. b. Use frequently occurring nouns and verbs. c. Form regular plural nouns orally by adding /s/ or /es/ (e.g., *dog, dogs; wish, wishes*). d. Understand and use question words (interrogatives) (e.g., *who, what, where, when, why, how*). e. Use the most frequently occurring prepositions (e.g., *to, from, in, out, on, off, for, of, by, with*). f. Produce and expand complete sentences in shared language activities.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Print all upper- and lowercase letters. b. Use common, proper, and possessive nouns. c. Use singular and plural nouns with matching verbs in basic sentences (e.g., *He hops; We hop*). d. Use personal, possessive, and indefinite pronouns (e.g., *I, me, my; they, them, their; anyone, everything*). e. Use verbs to convey a sense of past, present, and future (e.g., *Yesterday I walked home; Today I walk home; Tomorrow I will walk home*). f. Use frequently occurring adjectives. g. Use frequently occurring conjunctions (e.g., *and, but, or, so, because*). h. Use determiners (e.g., articles, demonstratives). i. Use frequently occurring prepositions (e.g., *during, beyond, toward*). j. Produce and expand complete simple and compound declarative, interrogative, imperative, and exclamatory sentences in response to prompts.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use collective nouns (e.g., *group*). b. Form and use frequently occurring irregular plural nouns (e.g., *feet, children, teeth, mice, fish*). c. Use reflexive pronouns (e.g., *myself, ourselves*). d. Form and use the past tense of frequently occurring irregular verbs (e.g., *sat, hid, told*). e. Use adjectives and adverbs, and choose between them depending on what is to be modified. f. Produce, expand, and rearrange complete simple and compound sentences (e.g., *The boy watched the movie; The little boy watched the movie; The action movie was watched by the little boy*).
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize the first word in a sentence and the pronoun *I*. b. Recognize and name end punctuation. c. Write a letter or letters for most consonant and short-vowel sounds (phonemes). d. Spell simple words phonetically, drawing on knowledge of sound-letter relationships.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize dates and names of people. b. Use end punctuation for sentences. c. Use commas in dates and to separate single words in a series. d. Use conventional spelling for words with common spelling patterns and for frequently occurring irregular words. e. Spell untaught words phonetically, drawing on phonemic awareness and spelling conventions.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize holidays, product names, and geographic names. b. Use commas in greetings and closings of letters. c. Use an apostrophe to form contractions and frequently occurring possessives. d. Generalize learned spelling patterns when writing words (e.g., cage → badge; boy → boil). e. Consult reference materials, including beginning dictionaries, as needed to check and correct spellings.

Language Standards K–5

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Kindergartners:	Grade 1 students:	Grade 2 students:
Knowledge of Language		
3. (Begins in grade 2)	3. (Begins in grade 2)	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Compare formal and informal uses of English.
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *kindergarten reading and content.* a. Identify new meanings for familiar words and apply them accurately (e.g., knowing *duck* is a bird and learning the verb *to duck*). b. Use the most frequently occurring inflections and affixes (e.g., *-ed, -s, re-, un-, pre-, -ful, -less*) as a clue to the meaning of an unknown word.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 1 reading and content*, choosing flexibly from an array of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Use frequently occurring affixes as a clue to the meaning of a word. c. Identify frequently occurring root words (e.g., *look*) and their inflectional forms (e.g., *looks, looked, looking*).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 2 reading and content*, choosing flexibly from an array of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Determine the meaning of the new word formed when a known prefix is added to a known word (e.g., *happy/unhappy, tell/retell*). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., *addition, additional*). d. Use knowledge of the meaning of individual words to predict the meaning of compound words (e.g., *birdhouse, lighthouse, housefly; bookshelf, notebook, bookmark*). e. Use glossaries and beginning dictionaries, both print and digital, to determine or clarify the meaning of words and phrases.
5. With guidance and support from adults, explore word relationships and nuances in word meanings. a. Sort common objects into categories (e.g., shapes, foods) to gain a sense of the concepts the categories represent. b. Demonstrate understanding of frequently occurring verbs and adjectives by relating them to their opposites (antonyms). c. Identify real-life connections between words and their use (e.g., note places at school that are *colorful*). d. Distinguish shades of meaning among verbs describing the same general action (e.g., *walk, march, strut, prance*) by acting out the meanings.	5. With guidance and support from adults, demonstrate understanding of word relationships and nuances in word meanings. a. Sort words into categories (e.g., colors, clothing) to gain a sense of the concepts the categories represent. b. Define words by category and by one or more key attributes (e.g., a *duck* is a bird that swims; a *tiger* is a large cat with stripes). c. Identify real-life connections between words and their use (e.g., note places at home that are *cozy*). d. Distinguish shades of meaning among verbs differing in manner (e.g., *look, peek, glance, stare, glare, scowl*) and adjectives differing in intensity (e.g., *large, gigantic*) by defining or choosing them or by acting out the meanings.	5. Demonstrate understanding of word relationships and nuances in word meanings. a. Identify real-life connections between words and their use (e.g., describe foods that are *spicy* or *juicy*). b. Distinguish shades of meaning among closely related verbs (e.g., *toss, throw, hurl*) and closely related adjectives (e.g., *thin, slender, skinny, scrawny*).
6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts.	6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using frequently occurring conjunctions to signal simple relationships (e.g., *because*).	6. Use words and phrases acquired through conversations, reading and being read to, and responding to texts, including using adjectives and adverbs to describe (e.g., *When other kids are happy that makes me happy*).

Language Standards K–5

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Grade 3 students:	Grade 4 students:	Grade 5 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of nouns, pronouns, verbs, adjectives, and adverbs in general and their functions in particular sentences. b. Form and use regular and irregular plural nouns. c. Use abstract nouns (e.g., *childhood*). d. Form and use regular and irregular verbs. e. Form and use the simple (e.g., *I walked; I walk; I will walk*) verb tenses. f. Ensure subject-verb and pronoun-antecedent agreement.* g. Form and use comparative and superlative adjectives and adverbs, and choose between them depending on what is to be modified. h. Use coordinating and subordinating conjunctions. i. Produce simple, compound, and complex sentences.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use relative pronouns (*who, whose, whom, which, that*) and relative adverbs (*where, when, why*). b. Form and use the progressive (e.g., *I was walking; I am walking; I will be walking*) verb tenses. c. Use modal auxiliaries (e.g., *can, may, must*) to convey various conditions. d. Order adjectives within sentences according to conventional patterns (e.g., *a small red bag* rather than *a red small bag*). e. Form and use prepositional phrases. f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.* g. Correctly use frequently confused words (e.g., *to, too, two; there, their*).*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of conjunctions, prepositions, and interjections in general and their function in particular sentences. b. Form and use the perfect (e.g., *I had walked; I have walked; I will have walked*) verb tenses. c. Use verb tense to convey various times, sequences, states, and conditions. d. Recognize and correct inappropriate shifts in verb tense.* e. Use correlative conjunctions (e.g., *either/or, neither/nor*).
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Capitalize appropriate words in titles. b. Use commas in addresses. c. Use commas and quotation marks in dialogue. d. Form and use possessives. e. Use conventional spelling for high-frequency and other studied words and for adding suffixes to base words (e.g., *sitting, smiled, cries, happiness*). f. Use spelling patterns and generalizations (e.g., word families, position-based spellings, syllable patterns, ending rules, meaningful word parts) in writing words. g. Consult reference materials, including beginning dictionaries, as needed to check and correct spellings.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use correct capitalization. b. Use commas and quotation marks to mark direct speech and quotations from a text. c. Use a comma before a coordinating conjunction in a compound sentence. d. Spell grade-appropriate words correctly, consulting references as needed.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation to separate items in a series.* b. Use a comma to separate an introductory element from the rest of the sentence. c. Use a comma to set off the words *yes* and *no* (e.g., *Yes, thank you*), to set off a tag question from the rest of the sentence (e.g., *It's true, isn't it?*), and to indicate direct address (e.g., *Is that you, Steve?*). d. Use underlining, quotation marks, or italics to indicate titles of works. e. Spell grade-appropriate words correctly, consulting references as needed.

Language Standards K–5

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Grade 3 students:	Grade 4 students:	Grade 5 students:
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose words and phrases for effect.* b. Recognize and observe differences between the conventions of spoken and written standard English.	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose words and phrases to convey ideas precisely.* b. Choose punctuation for effect.* c. Differentiate between contexts that call for formal English (e.g., presenting ideas) and situations where informal discourse is appropriate (e.g., small-group discussion).	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Expand, combine, and reduce sentences for meaning, reader/listener interest, and style. b. Compare and contrast the varieties of English (e.g., dialects, registers) used in stories, dramas, or poems.
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning word and phrases based on *grade 3 reading and content*, choosing flexibly from a range of strategies. a. Use sentence-level context as a clue to the meaning of a word or phrase. b. Determine the meaning of the new word formed when a known affix is added to a known word (e.g., *agreeable/disagreeable, comfortable/uncomfortable, care/careless, heat/preheat*). c. Use a known root word as a clue to the meaning of an unknown word with the same root (e.g., *company, companion*). d. Use glossaries or beginning dictionaries, both print and digital, to determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 4 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., definitions, examples, or restatements in text) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., *telegraph, photograph, autograph*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 5 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., cause/effect relationships and comparisons in text) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek and Latin affixes and roots as clues to the meaning of a word (e.g., *photograph, photosynthesis*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation and determine or clarify the precise meaning of key words and phrases.
5. Demonstrate understanding of word relationships and nuances in word meanings. a. Distinguish the literal and nonliteral meanings of words and phrases in context (e.g., *take steps*). b. Identify real-life connections between words and their use (e.g., describe people who are *friendly* or *helpful*). c. Distinguish shades of meaning among related words that describe states of mind or degrees of certainty (e.g., *knew, believed, suspected, heard, wondered*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Explain the meaning of simple similes and metaphors (e.g., *as pretty as a picture*) in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Demonstrate understanding of words by relating them to their opposites (antonyms) and to words with similar but not identical meanings (synonyms).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figurative language, including similes and metaphors, in context. b. Recognize and explain the meaning of common idioms, adages, and proverbs. c. Use the relationship between particular words (e.g., synonyms, antonyms, homographs) to better understand each of the words.
6. Acquire and use accurately grade-appropriate conversational, general academic, and domain-specific words and phrases, including those that signal spatial and temporal relationships (e.g., *After dinner that night we went looking for them*).	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal precise actions, emotions, or states of being (e.g., *quizzed, whined, stammered*) and that are basic to a particular topic (e.g., *wildlife, conservation,* and *endangered* when discussing animal preservation).	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases, including those that signal contrast, addition, and other logical relationships (e.g., *however, although, nevertheless, similarly, moreover, in addition*).

Staying on Topic Within a Grade and Across Grades: How to Build Knowledge Systematically in English Language Arts K–5

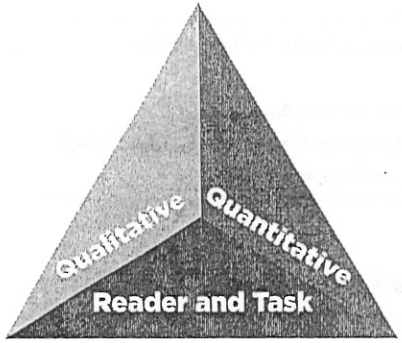
Building knowledge systematically in English language arts is like giving children various pieces of a puzzle in each grade that, over time, will form one big picture. At a curricular or instructional level, texts—within and across grade levels—need to be selected around topics or themes that systematically develop the knowledge base of students. Within a grade level, there should be an adequate number of titles on a single topic that would allow children to study that topic for a sustained period. The knowledge children have learned about particular topics in early grade levels should then be expanded and developed in subsequent grade levels to ensure an increasingly deeper understanding of these topics. Children in the upper elementary grades will generally be expected to read these texts independently and reflect on them in writing. However, children in the early grades (particularly K–2) should participate in rich, structured conversations with an adult in response to the written texts that are read aloud, orally comparing and contrasting as well as analyzing and synthesizing, in the manner called for by the *Standards*.

Preparation for reading complex informational texts should begin at the very earliest elementary school grades. What follows is one example that uses domain-specific nonfiction titles across grade levels to illustrate how curriculum designers and classroom teachers can infuse the English language arts block with rich, age-appropriate content knowledge and vocabulary in history/social studies, science, and the arts. Having students listen to informational read-alouds in the early grades helps lay the necessary foundation for students' reading and understanding of increasingly complex texts on their own in subsequent grades.

Exemplar Texts on a Topic Across Grades	K	1	2–3	4–5
The Human Body **Students can begin learning about the human body starting in kindergarten and then review and extend their learning during each subsequent grade.**	**The five senses and associated body parts** • *My Five Senses* by Aliki (1989) • *Hearing* by Maria Rius (1985) • *Sight* by Maria Rius (1985) • *Smell* by Maria Rius (1985) • *Taste* by Maria Rius (1985) • *Touch* by Maria Rius (1985) **Taking care of your body: Overview (hygiene, diet, exercise, rest)** • *My Amazing Body: A First Look at Health & Fitness* by Pat Thomas (2001) • *Get Up and Go!* by Nancy Carlson (2008) • *Go Wash Up* by Doering Tourville (2008) • *Sleep* by Paul Showers (1997) • *Fuel the Body* by Doering Tourville (2008)	**Introduction to the systems of the human body and associated body parts** • *Under Your Skin: Your Amazing Body* by Mick Manning (2007) • *Me and My Amazing Body* by Joan Sweeney (1999) • *The Human Body* by Gallimard Jeunesse (2007) • *The Busy Body Book* by Lizzy Rockwell (2008) • *First Encyclopedia of the Human Body* by Fiona Chandler (2004) **Taking care of your body: Germs, diseases, and preventing illness** • *Germs Make Me Sick* by Marilyn Berger (1995) • *Tiny Life on Your Body* by Christine Taylor-Butler (2005) • *Germ Stories* by Arthur Kornberg (2007) • *All About Scabs* by GenichiroYagu (1998)	**Digestive and excretory systems** • *What Happens to a Hamburger* by Paul Showers (1985) • *The Digestive System* by Christine Taylor-Butler (2008) • *The Digestive System* by Rebecca L. Johnson (2006) • *The Digestive System* by Kristin Petrie (2007) **Taking care of your body: Healthy eating and nutrition** • *Good Enough to Eat* by Lizzy Rockwell (1999) • *Showdown at the Food Pyramid* by Rex Barron (2004) **Muscular, skeletal, and nervous systems** • *The Mighty Muscular and Skeletal Systems* Crabtree Publishing (2009) • *Muscles* by Seymour Simon (1998) • *Bones* by Seymour Simon (1998) • *The Astounding Nervous System* Crabtree Publishing (2009) • *The Nervous System* by Joelle Riley (2004)	**Circulatory system** • *The Heart* by Seymour Simon (2006) • *The Heart and Circulation* by Carol Ballard (2005) • *The Circulatory System* by Kristin Petrie (2007) • *The Amazing Circulatory System* by John Burstein (2009) **Respiratory system** • *The Lungs* by Seymour Simon (2007) • *The Respiratory System* by Susan Glass (2004) • *The Respiratory System* by Kristin Petrie (2007) • *The Remarkable Respiratory System* by John Burstein (2009) **Endocrine system** • *The Endocrine System* by Rebecca Olien (2006) • *The Exciting Endocrine System* by John Burstein (2009)

Standard 10: Range, Quality, and Complexity of Student Reading K–5

Measuring Text Complexity: Three Factors



Qualitative evaluation of the text: Levels of meaning, structure, language conventionality and clarity, and knowledge demands

Quantitative evaluation of the text: Readability measures and other scores of text complexity

Matching reader to text and task: Reader variables (such as motivation, knowledge, and experiences) and task variables (such as purpose and the complexity generated by the task assigned and the questions posed)

Note: More detailed information on text complexity and how it is measured is contained in Appendix A.

Range of Text Types for K–5

Students in K–5 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

Literature			Informational Text
Stories	Dramas	Poetry	Literary Nonfiction and Historical, Scientific, and Technical Texts
Includes children's adventure stories, folktales, legends, fables, fantasy, realistic fiction, and myth	Includes staged dialogue and brief familiar scenes	Includes nursery rhymes and the subgenres of the narrative poem, limerick, and free verse poem	Includes biographies and autobiographies; books about history, social studies, science, and the arts; technical texts, including directions, forms, and information displayed in graphs, charts, or maps; and digital sources on a range of topics

** Read-along

Texts Illustrating the Complexity, Quality, and Range of Student Reading K–5

	Literature: Stories, Drama, Poetry	Informational Texts: Literary Nonfiction and Historical, Scientific, and Technical Texts
K*	*Over in the Meadow* by John Langstaff (traditional) (c1800)* *A Boy, a Dog, and a Frog* by Mercer Mayer (1967) *Pancakes for Breakfast* by Tomie DePaola (1978) *A Story, A Story* by Gail E. Haley (1970)* *Kitten's First Full Moon* by Kevin Henkes (2004)*	*My Five Senses* by Aliki (1962)** *Truck* by Donald Crews (1980) *I Read Signs* by Tana Hoban (1987) *What Do You Do With a Tail Like This?* by Steve Jenkins and Robin Page (2003)* *Amazing Whales!* by Sarah L. Thomson (2005)*
1*	"Mix a Pancake" by Christina G. Rossetti (1893)** *Mr. Popper's Penguins* by Richard Atwater (1938)* *Little Bear* by Else Holmelund Minarik, illustrated by Maurice Sendak (1957)** *Frog and Toad Together* by Arnold Lobel (1971)** *Hi! Fly Guy* by Tedd Arnold (2006)	*A Tree Is a Plant* by Clyde Robert Bulla, illustrated by Stacey Schuett (1960)** *Starfish* by Edith Thacher Hurd (1962) *Follow the Water from Brook to Ocean* by Arthur Dorros (1991)** *From Seed to Pumpkin* by Wendy Pfeffer, illustrated by James Graham Hale (2004)* *How People Learned to Fly* by Fran Hodgkins and True Kelley (2007)*
2–3	"Who Has Seen the Wind?" by Christina G. Rossetti (1893) *Charlotte's Web* by E. B. White (1952)* *Sarah, Plain and Tall* by Patricia MacLachlan (1985) *Tops and Bottoms* by Janet Stevens (1995) *Poppleton in Winter* by Cynthia Rylant, illustrated by Mark Teague (2001)	*A Medieval Feast* by Aliki (1983) *From Seed to Plant* by Gail Gibbons (1991) *The Story of Ruby Bridges* by Robert Coles (1995)* *A Drop of Water: A Book of Science and Wonder* by Walter Wick (1997) *Moonshot: The Flight of Apollo 11* by Brian Floca (2009)
4–5	*Alice's Adventures in Wonderland* by Lewis Carroll (1865) "Casey at the Bat" by Ernest Lawrence Thayer (1888) *The Black Stallion* by Walter Farley (1941) "Zlateh the Goat" by Isaac Bashevis Singer (1984) *Where the Mountain Meets the Moon* by Grace Lin (2009)	*Discovering Mars: The Amazing Story of the Red Planet* by Melvin Berger (1992) *Hurricanes: Earth's Mightiest Storms* by Patricia Lauber (1996) *A History of US* by Joy Hakim (2005) *Horses* by Seymour Simon (2006) *Quest for the Tree Kangaroo: An Expedition to the Cloud Forest of New Guinea* by Sy Montgomery (2006)

Note: Given space limitations, the illustrative texts listed above are meant only to show individual titles that are representative of a wide range of topics and genres. (See Appendix B for excerpts of these and other texts illustrative of K–5 text complexity, quality, and range.) At a curricular or instructional level, within and across grade levels, texts need to be selected around topics or themes that generate knowledge and allow students to study those topics or themes in depth. On the next page is an example of progressions of texts building knowledge across grade levels.

*Children at the kindergarten and grade 1 levels should be expected to read texts independently that have been specifically written to correlate to their reading level and their word knowledge. Many of the titles listed above are meant to supplement carefully structured independent reading with books to read along with a teacher or that are read aloud to students to build knowledge and cultivate a joy in reading.

Staying on Topic Within a Grade and Across Grades: How to Build Knowledge Systematically in English Language Arts K–5

Building knowledge systematically in English language arts is like giving children various pieces of a puzzle in each grade that, over time, will form one big picture. At a curricular or instructional level, texts—within and across grade levels—need to be selected around topics or themes that systematically develop the knowledge base of students. Within a grade level, there should be an adequate number of titles on a single topic that would allow children to study that topic for a sustained period. The knowledge children have learned about particular topics in early grade levels should then be expanded and developed in subsequent grade levels to ensure an increasingly deeper understanding of these topics. Children in the upper elementary grades will generally be expected to read these texts independently and reflect on them in writing. However, children in the early grades (particularly K–2) should participate in rich, structured conversations with an adult in response to the written texts that are read aloud, orally comparing and contrasting as well as analyzing and synthesizing, in the manner called for by the *Standards*.

Preparation for reading complex informational texts should begin at the very earliest elementary school grades. What follows is one example that uses domain-specific nonfiction titles across grade levels to illustrate how curriculum designers and classroom teachers can infuse the English language arts block with rich, age-appropriate content knowledge and vocabulary in history/social studies, science, and the arts. Having students listen to informational read-alouds in the early grades helps lay the necessary foundation for students' reading and understanding of increasingly complex texts on their own in subsequent grades.

Exemplar Texts on a Topic Across Grades	K	1	2–3	4–5
The Human Body **Students can begin learning about the human body starting in kindergarten and then review and extend their learning during each subsequent grade.**	**The five senses and associated body parts** • *My Five Senses* by Aliki (1989) • *Hearing* by Maria Rius (1985) • *Sight* by Maria Rius (1985) • *Smell* by Maria Rius (1985) • *Taste* by Maria Rius (1985) • *Touch* by Maria Rius (1985) **Taking care of your body: Overview (hygiene, diet, exercise, rest)** • *My Amazing Body: A First Look at Health & Fitness* by Pat Thomas (2001) • *Get Up and Go!* by Nancy Carlson (2008) • *Go Wash Up* by Doering Tourville (2008) • *Sleep* by Paul Showers (1997) • *Fuel the Body* by Doering Tourville (2008)	**Introduction to the systems of the human body and associated body parts** • *Under Your Skin: Your Amazing Body* by Mick Manning (2007) • *Me and My Amazing Body* by Joan Sweeney (1999) • *The Human Body* by Gallimard Jeunesse (2007) • *The Busy Body Book* by Lizzy Rockwell (2008) • *First Encyclopedia of the Human Body* by Fiona Chandler (2004) **Taking care of your body: Germs, diseases, and preventing illness** • *Germs Make Me Sick* by Marilyn Berger (1995) • *Tiny Life on Your Body* by Christine Taylor-Butler (2005) • *Germ Stories* by Arthur Kornberg (2007) • *All About Scabs* by GenichiroYagu (1998)	**Digestive and excretory systems** • *What Happens to a Hamburger* by Paul Showers (1985) • *The Digestive System* by Christine Taylor-Butler (2008) • *The Digestive System* by Rebecca L. Johnson (2006) • *The Digestive System* by Kristin Petrie (2007) **Taking care of your body: Healthy eating and nutrition** • *Good Enough to Eat* by Lizzy Rockwell (1999) • *Showdown at the Food Pyramid* by Rex Barron (2004) **Muscular, skeletal, and nervous systems** • *The Mighty Muscular and Skeletal Systems* Crabtree Publishing (2009) • *Muscles* by Seymour Simon (1998) • *Bones* by Seymour Simon (1998) • *The Astounding Nervous System* Crabtree Publishing (2009) • *The Nervous System* by Joelle Riley (2004)	**Circulatory system** • *The Heart* by Seymour Simon (2006) • *The Heart and Circulation* by Carol Ballard (2005) • *The Circulatory System* by Kristin Petrie (2007) • *The Amazing Circulatory System* by John Burstein (2009) **Respiratory system** • *The Lungs* by Seymour Simon (2007) • *The Respiratory System* by Susan Glass (2004) • *The Respiratory System* by Kristin Petrie (2007) • *The Remarkable Respiratory System* by John Burstein (2009) **Endocrine system** • *The Endocrine System* by Rebecca Olien (2006) • *The Exciting Endocrine System* by John Burstein (2009)

STANDARDS FOR

English Language Arts

6–12

Reading Standards for Literature 6–12

RL

The following standards offer a focus for instruction each year and help ensure that students gain adequate exposure to a range of texts and tasks. Rigor is also infused through the requirement that students read increasingly complex texts through the grades. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details		
1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
2. Determine a theme or central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.	2. Determine a theme or central idea of a text and analyze its development over the course of the text; provide an objective summary of the text.	2. Determine a theme or central idea of a text and analyze its development over the course of the text, including its relationship to the characters, setting, and plot; provide an objective summary of the text.
3. Describe how a particular story's or drama's plot unfolds in a series of episodes as well as how the characters respond or change as the plot moves toward a resolution.	3. Analyze how particular elements of a story or drama interact (e.g., how setting shapes the characters or plot).	3. Analyze how particular lines of dialogue or incidents in a story or drama propel the action, reveal aspects of a character, or provoke a decision.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of rhymes and other repetitions of sounds (e.g., alliteration) on a specific verse or stanza of a poem or section of a story or drama.	4. Determine the meaning of words and phrases as they are used in a text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
5. Analyze how a particular sentence, chapter, scene, or stanza fits into the overall structure of a text and contributes to the development of the theme, setting, or plot.	5. Analyze how a drama's or poem's form or structure (e.g., soliloquy, sonnet) contributes to its meaning.	5. Compare and contrast the structure of two or more texts and analyze how the differing structure of each text contributes to its meaning and style.
6. Explain how an author develops the point of view of the narrator or speaker in a text.	6. Analyze how an author develops and contrasts the points of view of different characters or narrators in a text.	6. Analyze how differences in the points of view of the characters and the audience or reader (e.g., created through the use of dramatic irony) create such effects as suspense or humor.

Reading Standards for Literature 6–12

RL

Grade 6 students:	Grade 7 students:	Grade 8 students:
Integration of Knowledge and Ideas		
7. Compare and contrast the experience of reading a story, drama, or poem to listening to or viewing an audio, video, or live version of the text, including contrasting what they "see" and "hear" when reading the text to what they perceive when they listen or watch.	7. Compare and contrast a written story, drama, or poem to its audio, filmed, staged, or multimedia version, analyzing the effects of techniques unique to each medium (e.g., lighting, sound, color, or camera focus and angles in a film).	7. Analyze the extent to which a filmed or live production of a story or drama stays faithful to or departs from the text or script, evaluating the choices made by the director or actors.
8. (Not applicable to literature)	8. (Not applicable to literature)	8. (Not applicable to literature)
9. Compare and contrast texts in different forms or genres (e.g., stories and poems; historical novels and fantasy stories) in terms of their approaches to similar themes and topics.	9. Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history.	9. Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literature, including stories, dramas, and poems, at the high end of grades 6–8 text complexity band independently and proficiently.

Reading Standards for Literature 6–12

RL

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details	
1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
2. Determine a theme or central idea of a text and analyze in detail its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.	2. Determine two or more themes or central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to produce a complex account; provide an objective summary of the text.
3. Analyze how complex characters (e.g., those with multiple or conflicting motivations) develop over the course of a text, interact with other characters, and advance the plot or develop the theme.	3. Analyze the impact of the author's choices regarding how to develop and relate elements of a story or drama (e.g., where a story is set, how the action is ordered, how the characters are introduced and developed).
Craft and Structure	
4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language evokes a sense of time and place; how it sets a formal or informal tone).	4. Determine the meaning of words and phrases as they are used in the text, including figurative and connotative meanings; analyze the impact of specific word choices on meaning and tone, including words with multiple meanings or language that is particularly fresh, engaging, or beautiful. (Include Shakespeare as well as other authors.)
5. Analyze how an author's choices concerning how to structure a text, order events within it (e.g., parallel plots), and manipulate time (e.g., pacing, flashbacks) create such effects as mystery, tension, or surprise.	5. Analyze how an author's choices concerning how to structure specific parts of a text (e.g., the choice of where to begin or end a story, the choice to provide a comedic or tragic resolution) contribute to its overall structure and meaning as well as its aesthetic impact.
6. Analyze a particular point of view or cultural experience reflected in a work of literature from outside the United States, drawing on a wide reading of world literature.	6. Analyze a case in which grasping point of view requires distinguishing what is directly stated in a text from what is really meant (e.g., satire, sarcasm, irony, or understatement).
Integration of Knowledge and Ideas	
7. Analyze the representation of a subject or a key scene in two different artistic mediums, including what is emphasized or absent in each treatment (e.g., Auden's "Musée des Beaux Arts" and Breughel's *Landscape with the Fall of Icarus*).	7. Analyze multiple interpretations of a story, drama, or poem (e.g., recorded or live production of a play or recorded novel or poetry), evaluating how each version interprets the source text. (Include at least one play by Shakespeare and one play by an American dramatist.)
8. (Not applicable to literature)	8. (Not applicable to literature)
9. Analyze how an author draws on and transforms source material in a specific work (e.g., how Shakespeare treats a theme or topic from Ovid or the Bible or how a later author draws on a play by Shakespeare).	9. Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics.
Range of Reading and Level of Text Complexity	
10. By the end of grade 9, read and comprehend literature, including stories, dramas, and poems, in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 10, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 11, read and comprehend literature, including stories, dramas, and poems, in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literature, including stories, dramas, and poems, at the high end of the grades 11–CCR text complexity band independently and proficiently.

Reading Standards for Informational Text 6–12

RI

Grade 6 students:	Grade 7 students:	Grade 8 students:
Key Ideas and Details		
1. Cite textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite several pieces of textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite the textual evidence that most strongly supports an analysis of what the text says explicitly as well as inferences drawn from the text.
2. Determine a central idea of a text and how it is conveyed through particular details; provide a summary of the text distinct from personal opinions or judgments.	2. Determine two or more central ideas in a text and analyze their development over the course of the text; provide an objective summary of the text.	2. Determine a central idea of a text and analyze its development over the course of the text, including its relationship to supporting ideas; provide an objective summary of the text.
3. Analyze in detail how a key individual, event, or idea is introduced, illustrated, and elaborated in a text (e.g., through examples or anecdotes).	3. Analyze the interactions between individuals, events, and ideas in a text (e.g., how ideas influence individuals or events, or how individuals influence ideas or events).	3. Analyze how a text makes connections among and distinctions between individuals, ideas, or events (e.g., through comparisons, analogies, or categories).
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of a specific word choice on meaning and tone.	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the impact of specific word choices on meaning and tone, including analogies or allusions to other texts.
5. Analyze how a particular sentence, paragraph, chapter, or section fits into the overall structure of a text and contributes to the development of the ideas.	5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to the development of the ideas.	5. Analyze in detail the structure of a specific paragraph in a text, including the role of particular sentences in developing and refining a key concept.
6. Determine an author's point of view or purpose in a text and explain how it is conveyed in the text.	6. Determine an author's point of view or purpose in a text and analyze how the author distinguishes his or her position from that of others.	6. Determine an author's point of view or purpose in a text and analyze how the author acknowledges and responds to conflicting evidence or viewpoints.
Integration of Knowledge and Ideas		
7. Integrate information presented in different media or formats (e.g., visually, quantitatively) as well as in words to develop a coherent understanding of a topic or issue.	7. Compare and contrast a text to an audio, video, or multimedia version of the text, analyzing each medium's portrayal of the subject (e.g., how the delivery of a speech affects the impact of the words).	7. Evaluate the advantages and disadvantages of using different mediums (e.g., print or digital text, video, multimedia) to present a particular topic or idea.
8. Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not.	8. Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims.	8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced.
9. Compare and contrast one author's presentation of events with that of another (e.g., a memoir written by and a biography on the same person).	9. Analyze how two or more authors writing about the same topic shape their presentations of key information by emphasizing different evidence or advancing different interpretations of facts.	9. Analyze a case in which two or more texts provide conflicting information on the same topic and identify where the texts disagree on matters of fact or interpretation.
Range of Reading and Level of Text Complexity		
10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction in the grades 6–8 text complexity band proficiently, with scaffolding as needed at the high end of the range.	10. By the end of the year, read and comprehend literary nonfiction at the high end of the grades 6–8 text complexity band independently and proficiently.

Reading Standards for Informational Text 6–12

RI

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details	
1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text.	1. Cite strong and thorough textual evidence to support analysis of what the text says explicitly as well as inferences drawn from the text, including determining where the text leaves matters uncertain.
2. Determine a central idea of a text and analyze its development over the course of the text, including how it emerges and is shaped and refined by specific details; provide an objective summary of the text.	2. Determine two or more central ideas of a text and analyze their development over the course of the text, including how they interact and build on one another to provide a complex analysis; provide an objective summary of the text.
3. Analyze how the author unfolds an analysis or series of ideas or events, including the order in which the points are made, how they are introduced and developed, and the connections that are drawn between them.	3. Analyze a complex set of ideas or sequence of events and explain how specific individuals, ideas, or events interact and develop over the course of the text.
Craft and Structure	
4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze the cumulative impact of specific word choices on meaning and tone (e.g., how the language of a court opinion differs from that of a newspaper).	4. Determine the meaning of words and phrases as they are used in a text, including figurative, connotative, and technical meanings; analyze how an author uses and refines the meaning of a key term or terms over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10).
5. Analyze in detail how an author's ideas or claims are developed and refined by particular sentences, paragraphs, or larger portions of a text (e.g., a section or chapter).	5. Analyze and evaluate the effectiveness of the structure an author uses in his or her exposition or argument, including whether the structure makes points clear, convincing, and engaging.
6. Determine an author's point of view or purpose in a text and analyze how an author uses rhetoric to advance that point of view or purpose.	6. Determine an author's point of view or purpose in a text in which the rhetoric is particularly effective, analyzing how style and content contribute to the power, persuasiveness, or beauty of the text.
Integration of Knowledge and Ideas	
7. Analyze various accounts of a subject told in different mediums (e.g., a person's life story in both print and multimedia), determining which details are emphasized in each account.	7. Integrate and evaluate multiple sources of information presented in different media or formats (e.g., visually, quantitatively) as well as in words in order to address a question or solve a problem.
8. Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning.	8. Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning (e.g., in U.S. Supreme Court majority opinions and dissents) and the premises, purposes, and arguments in works of public advocacy (e.g., *The Federalist*, presidential addresses).
9. Analyze seminal U.S. documents of historical and literary significance (e.g., Washington's Farewell Address, the Gettysburg Address, Roosevelt's Four Freedoms speech, King's "Letter from Birmingham Jail"), including how they address related themes and concepts.	9. Analyze seventeenth-, eighteenth-, and nineteenth-century foundational U.S. documents of historical and literary significance (including The Declaration of Independence, the Preamble to the Constitution, the Bill of Rights, and Lincoln's Second Inaugural Address) for their themes, purposes, and rhetorical features.
Range of Reading and Level of Text Complexity	
10. By the end of grade 9, read and comprehend literary nonfiction in the grades 9–10 text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 10, read and comprehend literary nonfiction at the high end of the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 11, read and comprehend literary nonfiction in the grades 11–CCR text complexity band proficiently, with scaffolding as needed at the high end of the range. By the end of grade 12, read and comprehend literary nonfiction at the high end of the grades 11–CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college- and career-ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to know how to combine elements of different kinds of writing—for example, to use narrative strategies within argument and explanation within narrative—to produce complex and nuanced writing. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline as well as the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it.

Writing Standards 6–12

W

The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. Each year in their writing, students should demonstrate increasing sophistication in all aspects of language use, from vocabulary and syntax to the development and organization of ideas, and they should address increasingly demanding content and sources. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* The expected growth in student writing ability is reflected both in the standards themselves and in the collection of annotated student writing samples in Appendix C.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes		
1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s) and organize the reasons and evidence clearly. b. Support claim(s) with clear reasons and relevant evidence, using credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to clarify the relationships among claim(s) and reasons. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from the argument presented.	1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s), acknowledge alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims with clear reasons and relevant evidence. a. Introduce claim(s), acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant evidence, using accurate, credible sources and demonstrating an understanding of the topic or text. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.
2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information, using strategies such as definition, classification, comparison/contrast, and cause/effect; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts to examine a topic and convey ideas, concepts, and information through the selection, organization, and analysis of relevant content. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.

Writing Standards 6–12

W

Grade 6 students:	Grade 7 students:	Grade 8 students:
Text Types and Purposes (continued)		
3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. d. Use precise words and phrases, relevant descriptive details, and sensory language to convey experiences and events. e. Provide a conclusion that follows from the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, and description, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence and signal shifts from one time frame or setting to another. d. Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. e. Provide a conclusion that follows from and reflects on the narrated experiences or events.	3. Write narratives to develop real or imagined experiences or events using effective technique, relevant descriptive details, and well-structured event sequences. a. Engage and orient the reader by establishing a context and point of view and introducing a narrator and/or characters; organize an event sequence that unfolds naturally and logically. b. Use narrative techniques, such as dialogue, pacing, description, and reflection, to develop experiences, events, and/or characters. c. Use a variety of transition words, phrases, and clauses to convey sequence, signal shifts from one time frame or setting to another, and show the relationships among experiences and events. d. Use precise words and phrases, relevant descriptive details, and sensory language to capture the action and convey experiences and events. e. Provide a conclusion that follows from and reflects on the narrated experiences or events.
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 6 on page 52.)	5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 7 on page 52.)	5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grade 8 on page 52.)
6. Use technology, including the Internet, to produce and publish writing as well as to interact and collaborate with others; demonstrate sufficient command of keyboarding skills to type a minimum of three pages in a single sitting.	6. Use technology, including the Internet, to produce and publish writing and link to and cite sources as well as to interact and collaborate with others, including linking to and citing sources.	6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas efficiently as well as to interact and collaborate with others.

Writing Standards 6–12

Grade 6 students:	Grade 7 students:	Grade 8 students:
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question, drawing on several sources and refocusing the inquiry when appropriate.	7. Conduct short research projects to answer a question, drawing on several sources and generating additional related, focused questions for further research and investigation.	7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.
8. Gather relevant information from multiple print and digital sources; assess the credibility of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and providing basic bibliographic information for sources.	8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 6 Reading standards* to literature (e.g., "Compare and contrast texts in different forms or genres [e.g., stories and poems; historical novels and fantasy stories] in terms of their approaches to similar themes and topics"). b. Apply *grade 6 Reading standards* to literary nonfiction (e.g., "Trace and evaluate the argument and specific claims in a text, distinguishing claims that are supported by reasons and evidence from claims that are not").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 7 Reading standards* to literature (e.g., "Compare and contrast a fictional portrayal of a time, place, or character and a historical account of the same period as a means of understanding how authors of fiction use or alter history"). b. Apply *grade 7 Reading standards* to literary nonfiction (e.g. "Trace and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient to support the claims").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grade 8 Reading standards* to literature (e.g., "Analyze how a modern work of fiction draws on themes, patterns of events, or character types from myths, traditional stories, or religious works such as the Bible, including describing how the material is rendered new"). b. Apply *grade 8 Reading standards* to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is sound and the evidence is relevant and sufficient; recognize when irrelevant evidence is introduced").
Range of Writing		
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.

Writing Standards 6–12

W

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes	
1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments to support claims in an analysis of substantive topics or texts, using valid reasoning and relevant and sufficient evidence. a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant evidence for each while pointing out the strengths and limitations of both in a manner that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from and supports the argument presented.
2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. a. Introduce a topic; organize complex ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use appropriate and varied transitions to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. a. Introduce a topic; organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use appropriate and varied transitions and syntax to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary, and techniques such as metaphor, simile, and analogy to manage the complexity of the topic. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).

Writing Standards 6–12

W

Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes (continued)	
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences. a. Engage and orient the reader by setting out a problem, situation, or observation, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events. b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters. c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole. d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters. e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative.	3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details, and well-structured event sequences. a. Engage and orient the reader by setting out a problem, situation, or observation and its significance, establishing one or multiple point(s) of view, and introducing a narrator and/or characters; create a smooth progression of experiences or events. b. Use narrative techniques, such as dialogue, pacing, description, reflection, and multiple plot lines, to develop experiences, events, and/or characters. c. Use a variety of techniques to sequence events so that they build on one another to create a coherent whole and build toward a particular tone and outcome (e.g., a sense of mystery, suspense, growth, or resolution). d. Use precise words and phrases, telling details, and sensory language to convey a vivid picture of the experiences, events, setting, and/or characters. e. Provide a conclusion that follows from and reflects on what is experienced, observed, or resolved over the course of the narrative.
Production and Distribution of Writing	
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience. (Grade-specific expectations for writing types are defined in standards 1–3 above.)
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 9–10 on page 54.)	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience. (Editing for conventions should demonstrate command of Language standards 1–3 up to and including grades 11–12 on page 54.)
6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge	
7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.

Writing Standards 6–12

W

Grades 9–10 students:	Grades 11–12 students:
Research to Build and Present Knowledge (continued)	
9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grades 9–10 Reading standards* to literature (e.g., "Analyze how an author draws on and transforms source material in a specific work [e.g., how Shakespeare treats a theme or topic from Ovid or the Bible or how a later author draws on a play by Shakespeare]"). b. Apply *grades 9–10 Reading standards* to literary nonfiction (e.g., "Delineate and evaluate the argument and specific claims in a text, assessing whether the reasoning is valid and the evidence is relevant and sufficient; identify false statements and fallacious reasoning").	9. Draw evidence from literary or informational texts to support analysis, reflection, and research. a. Apply *grades 11–12 Reading standards* to literature (e.g., "Demonstrate knowledge of eighteenth-, nineteenth- and early-twentieth-century foundational works of American literature, including how two or more texts from the same period treat similar themes or topics"). b. Apply *grades 11–12 Reading standards* to literary nonfiction (e.g., "Delineate and evaluate the reasoning in seminal U.S. texts, including the application of constitutional principles and use of legal reasoning [e.g., in U.S. Supreme Court Case majority opinions and dissents] and the premises, purposes, and arguments in works of public advocacy [e.g., *The Federalist*, presidential addresses]").
Range of Writing	
10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

College and Career Readiness Anchor Standards for Speaking and Listening

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Comprehension and Collaboration

1. Prepare for and participate effectively in a range of conversations and collaborations with diverse partners, building on others' ideas and expressing their own clearly and persuasively.
2. Integrate and evaluate information presented in diverse media and formats, including visually, quantitatively, and orally.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric.

Presentation of Knowledge and Ideas

4. Present information, findings, and supporting evidence such that listeners can follow the line of reasoning and the organization, development, and style are appropriate to task, purpose, and audience.
5. Make strategic use of digital media and visual displays of data to express information and enhance understanding of presentations.
6. Adapt speech to a variety of contexts and communicative tasks, demonstrating command of formal English when indicated or appropriate.

Note on range and content of student speaking and listening

To become college and career ready, students must have ample opportunities to take part in a variety of rich, structured conversations—as part of a whole class, in small groups, and with a partner—built around important content in various domains. They must be able to contribute appropriately to these conversations, to make comparisons and contrasts, and to analyze and synthesize a multitude of ideas in accordance with the standards of evidence appropriate to a particular discipline. Whatever their intended major or profession, high school graduates will depend heavily on their ability to listen attentively to others so that they are able to build on others' meritorious ideas while expressing their own clearly and persuasively.

New technologies have broadened and expanded the role that speaking and listening play in acquiring and sharing knowledge and have tightened their link to other forms of communication. The Internet has accelerated the speed at which connections between speaking, listening, reading, and writing can be made, requiring that students be ready to use these modalities nearly simultaneously. Technology itself is changing quickly, creating a new urgency for students to be adaptable in response to change.

Speaking and Listening Standards 6–12

SL

The following standards for grades 6–12 offer a focus for instruction in each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.*

Grade 6 students:	Grade 7 students:	Grade 8 students:
Comprehension and Collaboration		
1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 6 topics, texts, and issues*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or studied required material; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, set specific goals and deadlines, and define individual roles as needed. c. Pose and respond to specific questions with elaboration and detail by making comments that contribute to the topic, text, or issue under discussion. d. Review the key ideas expressed and demonstrate understanding of multiple perspectives through reflection and paraphrasing.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 7 topics, texts, and issues*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that elicit elaboration and respond to others' questions and comments with relevant observations and ideas that bring the discussion back on topic as needed. d. Acknowledge new information expressed by others and, when warranted, modify their own views.	1. Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 8 topics, texts, and issues*, building on others' ideas and expressing their own clearly. a. Come to discussions prepared, having read or researched material under study; explicitly draw on that preparation by referring to evidence on the topic, text, or issue to probe and reflect on ideas under discussion. b. Follow rules for collegial discussions and decision-making, track progress toward specific goals and deadlines, and define individual roles as needed. c. Pose questions that connect the ideas of several speakers and respond to others' questions and comments with relevant evidence, observations, and ideas. d. Acknowledge new information expressed by others, and, when warranted, qualify or justify their own views in light of the evidence presented.
2. Interpret information presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how it contributes to a topic, text, or issue under study.	2. Analyze the main ideas and supporting details presented in diverse media and formats (e.g., visually, quantitatively, orally) and explain how the ideas clarify a topic, text, or issue under study.	2. Analyze the purpose of information presented in diverse media and formats (e.g., visually, quantitatively, orally) and evaluate the motives (e.g., social, commercial, political) behind its presentation.
3. Delineate a speaker's argument and specific claims, distinguishing claims that are supported by reasons and evidence from claims that are not.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and the relevance and sufficiency of the evidence.	3. Delineate a speaker's argument and specific claims, evaluating the soundness of the reasoning and relevance and sufficiency of the evidence and identifying when irrelevant evidence is introduced.
Presentation of Knowledge and Ideas		
4. Present claims and findings, sequencing ideas logically and using pertinent descriptions, facts, and details to accentuate main ideas or themes; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with pertinent descriptions, facts, details, and examples; use appropriate eye contact, adequate volume, and clear pronunciation.	4. Present claims and findings, emphasizing salient points in a focused, coherent manner with relevant evidence, sound valid reasoning, and well-chosen details; use appropriate eye contact, adequate volume, and clear pronunciation.
5. Include multimedia components (e.g., graphics, images, music, sound) and visual displays in presentations to clarify information.	5. Include multimedia components and visual displays in presentations to clarify claims and findings and emphasize salient points.	5. Integrate multimedia and visual displays into presentations to clarify information, strengthen claims and evidence, and add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 6 Language standards 1 and 3 on page 52 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 7 Language standards 1 and 3 on page 52 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grade 8 Language standards 1 and 3 on page 52 for specific expectations.)

Speaking and Listening Standards 6–12

SL

The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Comprehension and Collaboration	
1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grades 9–10 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to set rules for collegial discussions and decision-making (e.g., informal consensus, taking votes on key issues, presentation of alternate views), clear goals and deadlines, and individual roles as needed. c. Propel conversations by posing and responding to questions that relate the current discussion to broader themes or larger ideas; actively incorporate others into the discussion; and clarify, verify, or challenge ideas and conclusions. d. Respond thoughtfully to diverse perspectives, summarize points of agreement and disagreement, and, when warranted, qualify or justify their own views and understanding and make new connections in light of the evidence and reasoning presented.	1. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively. a. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well-reasoned exchange of ideas. b. Work with peers to promote civil, democratic discussions and decision-making, set clear goals and deadlines, and establish individual roles as needed. c. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives. d. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task.
2. Integrate multiple sources of information presented in diverse media or formats (e.g., visually, quantitatively, orally) evaluating the credibility and accuracy of each source.	2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.
3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, identifying any fallacious reasoning or exaggerated or distorted evidence.	3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.
Presentation of Knowledge and Ideas	
4. Present information, findings, and supporting evidence clearly, concisely, and logically such that listeners can follow the line of reasoning and the organization, development, substance, and style are appropriate to purpose, audience, and task.	4. Present information, findings, and supporting evidence, conveying a clear and distinct perspective, such that listeners can follow the line of reasoning, alternative or opposing perspectives are addressed, and the organization, development, substance, and style are appropriate to purpose, audience, and a range of formal and informal tasks.
5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.	5. Make strategic use of digital media (e.g., textual, graphical, audio, visual, and interactive elements) in presentations to enhance understanding of findings, reasoning, and evidence and to add interest.
6. Adapt speech to a variety of contexts and tasks, demonstrating command of formal English when indicated or appropriate. (See grades 9–10 Language standards 1 and 3 on pages 54 for specific expectations.)	6. Adapt speech to a variety of contexts and tasks, demonstrating a command of formal English when indicated or appropriate. (See grades 11–12 Language standards 1 and 3 on page 54 for specific expectations.)

College and Career Readiness Anchor Standards for Language

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Conventions of Standard English

1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing.

Knowledge of Language

3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening.

Vocabulary Acquisition and Use

4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases by using context clues, analyzing meaningful word parts, and consulting general and specialized reference materials, as appropriate.
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings.
6. Acquire and use accurately a range of general academic and domain-specific words and phrases sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Note on range and content of student language use

To be college and career ready in language, students must have firm control over the conventions of standard English. At the same time, they must come to appreciate that language is as at least as much a matter of craft as of rules and be able to choose words, syntax, and punctuation to express themselves and achieve particular functions and rhetorical effects. They must also have extensive vocabularies, built through reading and study, enabling them to comprehend complex texts and engage in purposeful writing about and conversations around content. They need to become skilled in determining or clarifying the meaning of words and phrases they encounter, choosing flexibly from an array of strategies to aid them. They must learn to see an individual word as part of a network of other words—words, for example, that have similar denotations but different connotations. The inclusion of Language standards in their own strand should not be taken as an indication that skills related to conventions, effective language use, and vocabulary are unimportant to reading, writing, speaking, and listening; indeed, they are inseparable from such contexts.

Language Standards 6–12

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The following standards for grades 6–12 offer a focus for instruction each year to help ensure that students gain adequate mastery of a range of skills and applications. *Students advancing through the grades are expected to meet each year's grade-specific standards and retain or further develop skills and understandings mastered in preceding grades.* Beginning in grade 3, skills and understandings that are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking are marked with an asterisk (*). See the table on page 56 for a complete listing and Appendix A for an example of how these skills develop in sophistication.

Grade 6 students:	Grade 7 students:	Grade 8 students:
Conventions of Standard English		
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Ensure that pronouns are in the proper case (subjective, objective, possessive). b. Use intensive pronouns (e.g., *myself, ourselves*). c. Recognize and correct inappropriate shifts in pronoun number and person.* d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).* e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of phrases and clauses in general and their function in specific sentences. b. Choose among simple, compound, complex, and compound-complex sentences to signal differing relationships among ideas. c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.*	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Explain the function of verbals (gerunds, participles, infinitives) in general and their function in particular sentences. b. Form and use verbs in the active and passive voice. c. Form and use verbs in the indicative, imperative, interrogative, conditional, and subjunctive mood. d. Recognize and correct inappropriate shifts in verb voice and mood.*
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.* b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a comma to separate coordinate adjectives (e.g., *It was a fascinating, enjoyable movie* but not *He wore an old[,] green shirt*). b. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use punctuation (comma, ellipsis, dash) to indicate a pause or break. b. Use an ellipsis to indicate an omission. c. Spell correctly.
Knowledge of Language		
3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Vary sentence patterns for meaning, reader/listener interest, and style.* b. Maintain consistency in style and tone.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.*	3. Use knowledge of language and its conventions when writing, speaking, reading, or listening. a. Use verbs in the active and passive voice and in the conditional and subjunctive mood to achieve particular effects (e.g., emphasizing the actor or the action; expressing uncertainty or describing a state contrary to fact).

Language Standards 6–12

Grade 6 students:	Grade 7 students:	Grade 8 students:
Vocabulary Acquisition and Use		
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 6 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *audience, auditory, audible*). c. Consult reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grade 7 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *belligerent, bellicose, rebel*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words or phrases based on *grade 8 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence or paragraph; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Use common, grade-appropriate Greek or Latin affixes and roots as clues to the meaning of a word (e.g., *precede, recede, secede*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning or its part of speech. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., personification) in context. b. Use the relationship between particular words (e.g., cause/effect, part/whole, item/category) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *stingy, scrimping, economical, unwasteful, thrifty*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., literary, biblical, and mythological allusions) in context. b. Use the relationship between particular words (e.g., synonym/antonym, analogy) to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *refined, respectful, polite, diplomatic, condescending*).	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g. verbal irony, puns) in context. b. Use the relationship between particular words to better understand each of the words. c. Distinguish among the connotations (associations) of words with similar denotations (definitions) (e.g., *bullheaded, willful, firm, persistent, resolute*).
6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately grade-appropriate general academic and domain-specific words and phrases; gather vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Standards 6–12

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The CCR anchor standards and high school grade-specific standards work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 9–10 students:	Grades 11–12 students:
Conventions of Standard English	
1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Use parallel structure.* b. Use various types of phrases (noun, verb, adjectival, adverbial, participial, prepositional, absolute) and clauses (independent, dependent; noun, relative, adverbial) to convey specific meanings and add variety and interest to writing or presentations.	1. Demonstrate command of the conventions of standard English grammar and usage when writing or speaking. a. Apply the understanding that usage is a matter of convention, can change over time, and is sometimes contested. b. Resolve issues of complex or contested usage, consulting references (e.g., *Merriam-Webster's Dictionary of English Usage, Garner's Modern American Usage*) as needed.
2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Use a semicolon (and perhaps a conjunctive adverb) to link two or more closely related independent clauses. b. Use a colon to introduce a list or quotation. c. Spell correctly.	2. Demonstrate command of the conventions of standard English capitalization, punctuation, and spelling when writing. a. Observe hyphenation conventions. b. Spell correctly.
Knowledge of Language	
3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening. a. Write and edit work so that it conforms to the guidelines in a style manual (e.g., *MLA Handbook*, Turabian's *Manual for Writers*) appropriate for the discipline and writing type.	3. Apply knowledge of language to understand how language functions in different contexts, to make effective choices for meaning or style, and to comprehend more fully when reading or listening. a. Vary syntax for effect, consulting references (e.g., Tufte's *Artful Sentences*) for guidance as needed; apply an understanding of syntax to the study of complex texts when reading.

Language Standards 6–12

Grades 9–10 students:	Grades 11–12 students:
Vocabulary Acquisition and Use	
4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grades 9–10 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., *analyze, analysis, analytical; advocate, advocacy*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, or its etymology. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).	4. Determine or clarify the meaning of unknown and multiple-meaning words and phrases based on *grades 11–12 reading and content*, choosing flexibly from a range of strategies. a. Use context (e.g., the overall meaning of a sentence, paragraph, or text; a word's position or function in a sentence) as a clue to the meaning of a word or phrase. b. Identify and correctly use patterns of word changes that indicate different meanings or parts of speech (e.g., *conceive, conception, conceivable*). c. Consult general and specialized reference materials (e.g., dictionaries, glossaries, thesauruses), both print and digital, to find the pronunciation of a word or determine or clarify its precise meaning, its part of speech, its etymology, or its standard usage. d. Verify the preliminary determination of the meaning of a word or phrase (e.g., by checking the inferred meaning in context or in a dictionary).
5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., euphemism, oxymoron) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.	5. Demonstrate understanding of figurative language, word relationships, and nuances in word meanings. a. Interpret figures of speech (e.g., hyperbole, paradox) in context and analyze their role in the text. b. Analyze nuances in the meaning of words with similar denotations.
6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.	6. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level; demonstrate independence in gathering vocabulary knowledge when considering a word or phrase important to comprehension or expression.

Language Progressive Skills, by Grade

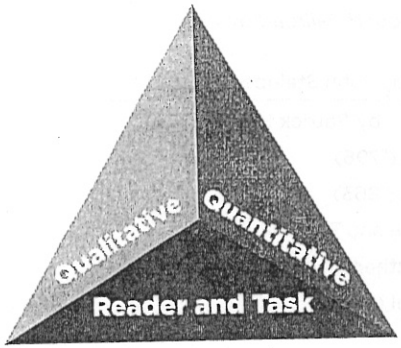
The following skills, marked with an asterisk (*) in Language standards 1–3, are particularly likely to require continued attention in higher grades as they are applied to increasingly sophisticated writing and speaking.

Standard	Grade(s)							
	3	4	5	6	7	8	9–10	11–12
L.3.1f. Ensure subject-verb and pronoun-antecedent agreement.								
L.3.3a. Choose words and phrases for effect.								
L.4.1f. Produce complete sentences, recognizing and correcting inappropriate fragments and run-ons.								
L.4.1g. Correctly use frequently confused words (e.g., *to/too/two; there/their*).								
L.4.3a. Choose words and phrases to convey ideas precisely.*								
L.4.3b. Choose punctuation for effect.								
L.5.1d. Recognize and correct inappropriate shifts in verb tense.								
L.5.2a. Use punctuation to separate items in a series.†								
L.6.1c. Recognize and correct inappropriate shifts in pronoun number and person.								
L.6.1d. Recognize and correct vague pronouns (i.e., ones with unclear or ambiguous antecedents).								
L.6.1e. Recognize variations from standard English in their own and others' writing and speaking, and identify and use strategies to improve expression in conventional language.								
L.6.2a. Use punctuation (commas, parentheses, dashes) to set off nonrestrictive/parenthetical elements.‡								
L.6.3a. Vary sentence patterns for meaning, reader/listener interest, and style.‡								
L.6.3b. Maintain consistency in style and tone.								
L.7.1c. Place phrases and clauses within a sentence, recognizing and correcting misplaced and dangling modifiers.								
L.7.3a. Choose language that expresses ideas precisely and concisely, recognizing and eliminating wordiness and redundancy.								
L.8.1d. Recognize and correct inappropriate shifts in verb voice and mood.								
L.9–10.1a. Use parallel structure.								

* Subsumed by L.7.3a
† Subsumed by L.9–10.1a
‡ Subsumed by L.11–12.3a

Standard 10: Range, Quality, and Complexity of Student Reading 6–12

Measuring Text Complexity: Three Factors



Qualitative evaluation of the text: Levels of meaning, structure, language conventionality and clarity, and knowledge demands

Quantitative evaluation of the text: Readability measures and other scores of text complexity

Matching reader to text and task: Reader variables (such as motivation, knowledge, and experiences) and task variables (such as purpose and the complexity generated by the task assigned and the questions posed)

Note: More detailed information on text complexity and how it is measured is contained in Appendix A.

Range of Text Types for 6–12

Students in grades 6–12 apply the Reading standards to the following range of text types, with texts selected from a broad range of cultures and periods.

Literature			Informational Text
Stories	Drama	Poetry	Literary Nonfiction
Includes the subgenres of adventure stories, historical fiction, mysteries, myths, science fiction, realistic fiction, allegories, parodies, satire, and graphic novels	Includes one-act and multi-act plays, both in written form and on film	Includes the subgenres of narrative poems, lyrical poems, free verse poems, sonnets, odes, ballads, and epics	Includes the subgenres of exposition, argument, and functional text in the form of personal essays, speeches, opinion pieces, essays about art or literature, biographies, memoirs, journalism, and historical, scientific, technical, or economic accounts (including digital sources) written for a broad audience

Texts Illustrating the Complexity, Quality, and Range of Student Reading 6–12

	Literature: Stories, Dramas, Poetry	Informational Texts: Literary Nonfiction
6–8	▪ *Little Women* by Louisa May Alcott (1869) ▪ *The Adventures of Tom Sawyer* by Mark Twain (1876) ▪ "The Road Not Taken" by Robert Frost (1915) ▪ *The Dark Is Rising* by Susan Cooper (1973) ▪ *Dragonwings* by Laurence Yep (1975) ▪ *Roll of Thunder, Hear My Cry* by Mildred Taylor (1976)	▪ "Letter on Thomas Jefferson" by John Adams (1776) ▪ *Narrative of the Life of Frederick Douglass, an American Slave* by Frederick Douglass (1845) ▪ "Blood, Toil, Tears and Sweat: Address to Parliament on May 13th, 1940" by Winston Churchill (1940) ▪ *Harriet Tubman: Conductor on the Underground Railroad* by Ann Petry (1955) ▪ *Travels with Charley: In Search of America* by John Steinbeck (1962)
9–10	▪ *The Tragedy of Macbeth* by William Shakespeare (1592) ▪ "Ozymandias" by Percy Bysshe Shelley (1817) ▪ "The Raven" by Edgar Allan Poe (1845) ▪ "The Gift of the Magi" by O. Henry (1906) ▪ *The Grapes of Wrath* by John Steinbeck (1939) ▪ *Fahrenheit 451* by Ray Bradbury (1953) ▪ *The Killer Angels* by Michael Shaara (1975)	▪ "Speech to the Second Virginia Convention" by Patrick Henry (1775) ▪ "Farewell Address" by George Washington (1796) ▪ "Gettysburg Address" by Abraham Lincoln (1863) ▪ "State of the Union Address" by Franklin Delano Roosevelt (1941) ▪ "Letter from Birmingham Jail" by Martin Luther King, Jr. (1964) ▪ "Hope, Despair and Memory" by Elie Wiesel (1997)
11–CCR	▪ "Ode on a Grecian Urn" by John Keats (1820) ▪ *Jane Eyre* by Charlotte Brontë (1848) ▪ "Because I Could Not Stop for Death" by Emily Dickinson (1890) ▪ *The Great Gatsby* by F. Scott Fitzgerald (1925) ▪ *Their Eyes Were Watching God* by Zora Neale Hurston (1937) ▪ *A Raisin in the Sun* by Lorraine Hansberry (1959) ▪ *The Namesake* by Jhumpa Lahiri (2003)	▪ *Common Sense* by Thomas Paine (1776) ▪ *Walden* by Henry David Thoreau (1854) ▪ "Society and Solitude" by Ralph Waldo Emerson (1857) ▪ "The Fallacy of Success" by G. K. Chesterton (1909) ▪ *Black Boy* by Richard Wright (1945) ▪ "Politics and the English Language" by George Orwell (1946) ▪ "Take the Tortillas Out of Your Poetry" by Rudolfo Anaya (1995)

Note: Given space limitations, the illustrative texts listed above are meant only to show individual titles that are representative of a range of topics and genres. (See Appendix B for excerpts of these and other texts illustrative of grades 6–12 text complexity, quality, and range.) At a curricular or instructional level, within and across grade levels, texts need to be selected around topics or themes that generate knowledge and allow students to study those topics or themes in depth.

SECTION 3

Common Core State Standards for Mathematics

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Introduction

Toward greater focus and coherence

> *Mathematics experiences in early childhood settings should concentrate on (1) number (which includes whole number, operations, and relations) and (2) geometry, spatial relations, and measurement, with more mathematics learning time devoted to number than to other topics. Mathematical process goals should be integrated in these content areas.*
>
> — Mathematics Learning in Early Childhood, National Research Council, 2009

> *The composite standards [of Hong Kong, Korea and Singapore] have a number of features that can inform an international benchmarking process for the development of K–6 mathematics standards in the U.S. First, the composite standards concentrate the early learning of mathematics on the number, measurement, and geometry strands with less emphasis on data analysis and little exposure to algebra. The Hong Kong standards for grades 1–3 devote approximately half the targeted time to numbers and almost all the time remaining to geometry and measurement.*
>
> — Ginsburg, Leinwand and Decker, 2009

> *Because the mathematics concepts in [U.S.] textbooks are often weak, the presentation becomes more mechanical than is ideal. We looked at both traditional and non-traditional textbooks used in the US and found this conceptual weakness in both.*
>
> — Ginsburg et al., 2005

> *There are many ways to organize curricula. The challenge, now rarely met, is to avoid those that distort mathematics and turn off students.*
>
> — Steen, 2007

For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order to improve mathematics achievement in this country. To deliver on the promise of common standards, the standards must address the problem of a curriculum that is "a mile wide and an inch deep." These Standards are a substantial answer to that challenge.

It is important to recognize that "fewer standards" are no substitute for focused standards. Achieving "fewer standards" would be easy to do by resorting to broad, general statements. Instead, these Standards aim for clarity and specificity.

Assessing the coherence of a set of standards is more difficult than assessing their focus. William Schmidt and Richard Houang (2002) have said that content standards and curricula are coherent if they are:

> *articulated over time as a sequence of topics and performances that are logical and reflect, where appropriate, the sequential or hierarchical nature of the disciplinary content from which the subject matter derives. That is, what and how students are taught should reflect not only the topics that fall within a certain academic discipline, **but also the key ideas** that determine how knowledge is organized and generated within that discipline. This implies*

> *that to be coherent, a set of content standards must evolve from particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures associated with whole numbers and fractions) to deeper structures inherent in the discipline. These deeper structures then serve as a means for connecting the particulars (such as an understanding of the rational number system and its properties).* (emphasis added)

These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.

In addition, the "sequence of topics and performances" that is outlined in a body of mathematics standards must also respect what is known about how students learn. As Confrey (2007) points out, developing "sequenced obstacles and challenges for students...absent the insights about meaning that derive from careful study of learning, would be unfortunate and unwise." In recognition of this, the development of these Standards began with research-based learning progressions detailing what is known today about how students' mathematical knowledge, skill, and understanding develop over time.

Understanding mathematics

These Standards define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards set grade-specific standards but do not define the intervention methods or materials necessary to support students who are well below or well above grade-level expectations. It is also beyond the scope of the Standards to define the full range of supports appropriate for English language learners and for students with special needs. At the same time, all students must have the opportunity to learn and meet the same high standards if they are to access the knowledge and skills necessary in their post-school lives. The Standards should be read as allowing for the widest possible range of students to participate fully from the outset, along with appropriate accommodations to ensure maximum participaton of students with special education needs. For example, for students with disabilities reading should allow for use of Braille, screen reader technology, or other assistive devices, while writing should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, speaking and listening should be interpreted broadly to include sign language. No set of grade-specific standards can fully reflect the great variety in abilities, needs, learning rates, and achievement levels of students in any given classroom. However, the Standards do provide clear signposts along the way to the goal of college and career readiness for all students.

The Standards begin on page 6 with eight Standards for Mathematical Practice.

How to read the grade level standards

Standards define what students should understand and be able to do.

Clusters are groups of related standards. Note that standards from different clusters may sometimes be closely related, because mathematics is a connected subject.

Domains are larger groups of related standards. Standards from different domains may sometimes be closely related.

Domain

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

Standard

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Cluster

These Standards do not dictate curriculum or teaching methods. For example, just because topic A appears before topic B in the standards for a given grade, it does not necessarily mean that topic A must be taught before topic B. A teacher might prefer to teach topic B before topic A, or might choose to highlight connections by teaching topic A and topic B at the same time. Or, a teacher might prefer to teach a topic of his or her own choosing that leads, as a byproduct, to students reaching the standards for topics A and B.

What students can learn at any particular grade level depends upon what they have learned before. Ideally then, each standard in this document might have been phrased in the form, "Students who already know ... should next come to learn ...." But at present this approach is unrealistic—not least because existing education research cannot specify all such learning pathways. Of necessity therefore, grade placements for specific topics have been made on the basis of state and international comparisons and the collective experience and collective professional judgment of educators, researchers and mathematicians. One promise of common state standards is that over time they will allow research on learning progressions to inform and improve the design of standards to a much greater extent than is possible today. Learning opportunities will continue to vary across schools and school systems, and educators should make every effort to meet the needs of individual students based on their current understanding.

These Standards are not intended to be new names for old ways of doing business. They are a call to take the next step. It is time for states to work together to build on lessons learned from two decades of standards based reforms. It is time to recognize that standards are not just promises to our children, but promises we intend to keep.

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Mathematics | Kindergarten

In Kindergarten, instructional time should focus on two critical areas: (1) representing, relating, and operating on whole numbers, initially with sets of objects; (2) describing shapes and space. More learning time in Kindergarten should be devoted to number than to other topics.

(1) Students use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set; counting out a given number of objects; comparing sets or numerals; and modeling simple joining and separating situations with sets of objects, or eventually with equations such as 5 + 2 = 7 and 7 – 2 = 5. (Kindergarten students should see addition and subtraction equations, and student writing of equations in kindergarten is encouraged, but it is not required.) Students choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the cardinalities of small sets of objects, counting and producing sets of given sizes, counting the number of objects in combined sets, or counting the number of objects that remain in a set after some are taken away.

(2) Students describe their physical world using geometric ideas (e.g., shape, orientation, spatial relations) and vocabulary. They identify, name, and describe basic two-dimensional shapes, such as squares, triangles, circles, rectangles, and hexagons, presented in a variety of ways (e.g., with different sizes and orientations), as well as three-dimensional shapes such as cubes, cones, cylinders, and spheres. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

Grade K Overview

Counting and Cardinality

- Know number names and the count sequence.
- Count to tell the number of objects.
- Compare numbers.

Operations and Algebraic Thinking

- Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

Number and Operations in Base Ten

- Work with numbers 11–19 to gain foundations for place value.

Measurement and Data

- Describe and compare measurable attributes.
- Classify objects and count the number of objects in categories.

Geometry

- Identify and describe shapes.
- Analyze, compare, create, and compose shapes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Counting and Cardinality — K.CC

Know number names and the count sequence.

1. Count to 100 by ones and by tens.
2. Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
3. Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Count to tell the number of objects.

4. Understand the relationship between numbers and quantities; connect counting to cardinality.
   a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
   b. Understand that the last number name said tells the number of objects counted. The number of objects is the same regardless of their arrangement or the order in which they were counted.
   c. Understand that each successive number name refers to a quantity that is one larger.
5. Count to answer "how many?" questions about as many as 20 things arranged in a line, a rectangular array, or a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

Compare numbers.

6. Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group, e.g., by using matching and counting strategies.[1]
7. Compare two numbers between 1 and 10 presented as written numerals.

Operations and Algebraic Thinking — K.OA

Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.

1. Represent addition and subtraction with objects, fingers, mental images, drawings[2], sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.
2. Solve addition and subtraction word problems, and add and subtract within 10, e.g., by using objects or drawings to represent the problem.
3. Decompose numbers less than or equal to 10 into pairs in more than one way, e.g., by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).
4. For any number from 1 to 9, find the number that makes 10 when added to the given number, e.g., by using objects or drawings, and record the answer with a drawing or equation.
5. Fluently add and subtract within 5.

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[1]Include groups with up to ten objects.
[2]Drawings need not show details, but should show the mathematics in the problem. (This applies wherever drawings are mentioned in the Standards.)

Number and Operations in Base Ten — K.NBT

Work with numbers 11–19 to gain foundations for place value.

1. Compose and decompose numbers from 11 to 19 into ten ones and some further ones, e.g., by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.

Measurement and Data — K.MD

Describe and compare measurable attributes.

1. Describe measurable attributes of objects, such as length or weight. Describe several measurable attributes of a single object.
2. Directly compare two objects with a measurable attribute in common, to see which object has "more of"/"less of" the attribute, and describe the difference. *For example, directly compare the heights of two children and describe one child as taller/shorter.*

Classify objects and count the number of objects in each category.

3. Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.[3]

Geometry — K.G

Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).

1. Describe objects in the environment using names of shapes, and describe the relative positions of these objects using terms such as *above*, *below*, *beside*, *in front of*, *behind*, and *next to*.
2. Correctly name shapes regardless of their orientations or overall size.
3. Identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Analyze, compare, create, and compose shapes.

4. Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners") and other attributes (e.g., having sides of equal length).
5. Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes.
6. Compose simple shapes to form larger shapes. *For example, "Can you join these two triangles with full sides touching to make a rectangle?"*

[3]Limit category counts to be less than or equal to 10.

Mathematics | Grade 1

In Grade 1, instructional time should focus on four critical areas: (1) developing understanding of addition, subtraction, and strategies for addition and subtraction within 20; (2) developing understanding of whole number relationships and place value, including grouping in tens and ones; (3) developing understanding of linear measurement and measuring lengths as iterating length units; and (4) reasoning about attributes of, and composing and decomposing geometric shapes.

(1) Students develop strategies for adding and subtracting whole numbers based on their prior work with small numbers. They use a variety of models, including discrete objects and length-based models (e.g., cubes connected to form lengths), to model add-to, take-from, put-together, take-apart, and compare situations to develop meaning for the operations of addition and subtraction, and to develop strategies to solve arithmetic problems with these operations. Students understand connections between counting and addition and subtraction (e.g., adding two is the same as counting on two). They use properties of addition to add whole numbers and to create and use increasingly sophisticated strategies based on these properties (e.g., "making tens") to solve addition and subtraction problems within 20. By comparing a variety of solution strategies, children build their understanding of the relationship between addition and subtraction.

(2) Students develop, discuss, and use efficient, accurate, and generalizable methods to add within 100 and subtract multiples of 10. They compare whole numbers (at least to 100) to develop understanding of and solve problems involving their relative sizes. They think of whole numbers between 10 and 100 in terms of tens and ones (especially recognizing the numbers 11 to 19 as composed of a ten and some ones). Through activities that build number sense, they understand the order of the counting numbers and their relative magnitudes.

(3) Students develop an understanding of the meaning and processes of measurement, including underlying concepts such as iterating (the mental activity of building up the length of an object with equal-sized units) and the transitivity principle for indirect measurement.[1]

(4) Students compose and decompose plane or solid figures (e.g., put two triangles together to make a quadrilateral) and build understanding of part-whole relationships as well as the properties of the original and composite shapes. As they combine shapes, they recognize them from different perspectives and orientations, describe their geometric attributes, and determine how they are alike and different, to develop the background for measurement and for initial understandings of properties such as congruence and symmetry.

[1]Students should apply the principle of transitivity of measurement to make indirect comparisons, but they need not use this technical term.

Grade 1 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Understand and apply properties of operations and the relationship between addition and subtraction.
- Add and subtract within 20.
- Work with addition and subtraction equations.

Number and Operations in Base Ten

- Extend the counting sequence.
- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure lengths indirectly and by iterating length units.
- Tell and write time.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 1.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.[2]

2. Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.

Understand and apply properties of operations and the relationship between addition and subtraction.

3. Apply properties of operations as strategies to add and subtract.[3] *Examples: If 8 + 3 = 11 is known, then 3 + 8 = 11 is also known. (Commutative property of addition.) To add 2 + 6 + 4, the second two numbers can be added to make a ten, so 2 + 6 + 4 = 2 + 10 = 12. (Associative property of addition.)*

4. Understand subtraction as an unknown-addend problem. *For example, subtract 10 – 8 by finding the number that makes 10 when added to 8.*

Add and subtract within 20.

5. Relate counting to addition and subtraction (e.g., by counting on 2 to add 2).

6. Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); using the relationship between addition and subtraction (e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and creating equivalent but easier or known sums (e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).

Work with addition and subtraction equations.

7. Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. *For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 – 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.*

8. Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11, 5 = [] – 3, 6 + 6 = [].*

Number and Operations in Base Ten 1.NBT

Extend the counting sequence.

1. Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects with a written numeral.

Understand place value.

2. Understand that the two digits of a two-digit number represent amounts of tens and ones. Understand the following as special cases:

   a. 10 can be thought of as a bundle of ten ones — called a "ten."

   b. The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.

   c. The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens (and 0 ones).

[2]See Glossary, Table 1.
[3]Students need not use formal terms for these properties.

3. Compare two two-digit numbers based on meanings of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.

Use place value understanding and properties of operations to add and subtract.

4. Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used. Understand that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.
5. Given a two-digit number, mentally find 10 more or 10 less than the number, without having to count; explain the reasoning used.
6. Subtract multiples of 10 in the range 10-90 from multiples of 10 in the range 10-90 (positive or zero differences), using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Measurement and Data 1.MD

Measure lengths indirectly and by iterating length units.

1. Order three objects by length; compare the lengths of two objects indirectly by using a third object.
2. Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. *Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.*

Tell and write time.

3. Tell and write time in hours and half-hours using analog and digital clocks.

Represent and interpret data.

4. Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

Geometry 1.G

Reason with shapes and their attributes.

1. Distinguish between defining attributes (e.g., triangles are closed and three-sided) versus non-defining attributes (e.g., color, orientation, overall size); build and draw shapes to possess defining attributes.
2. Compose two-dimensional shapes (rectangles, squares, trapezoids, triangles, half-circles, and quarter-circles) or three-dimensional shapes (cubes, right rectangular prisms, right circular cones, and right circular cylinders) to create a composite shape, and compose new shapes from the composite shape.[4]
3. Partition circles and rectangles into two and four equal shares, describe the shares using the words *halves*, *fourths*, and *quarters*, and use the phrases *half of*, *fourth of*, and *quarter of*. Describe the whole as two of, or four of the shares. Understand for these examples that decomposing into more equal shares creates smaller shares.

[4]Students do not need to learn formal names such as "right rectangular prism."

Mathematics | Grade 2

In Grade 2, instructional time should focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes.

(1) Students extend their understanding of the base-ten system. This includes ideas of counting in fives, tens, and multiples of hundreds, tens, and ones, as well as number relationships involving these units, including comparing. Students understand multi-digit numbers (up to 1000) written in base-ten notation, recognizing that the digits in each place represent amounts of thousands, hundreds, tens, or ones (e.g., 853 is 8 hundreds + 5 tens + 3 ones).

(2) Students use their understanding of addition to develop fluency with addition and subtraction within 100. They solve problems within 1000 by applying their understanding of models for addition and subtraction, and they develop, discuss, and use efficient, accurate, and generalizable methods to compute sums and differences of whole numbers in base-ten notation, using their understanding of place value and the properties of operations. They select and accurately apply methods that are appropriate for the context and the numbers involved to mentally calculate sums and differences for numbers with only tens or only hundreds.

(3) Students recognize the need for standard units of measure (centimeter and inch) and they use rulers and other measurement tools with the understanding that linear measure involves an iteration of units. They recognize that the smaller the unit, the more iterations they need to cover a given length.

(4) Students describe and analyze shapes by examining their sides and angles. Students investigate, describe, and reason about decomposing and combining shapes to make other shapes. Through building, drawing, and analyzing two- and three-dimensional shapes, students develop a foundation for understanding area, volume, congruence, similarity, and symmetry in later grades.

Grade 2 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving addition and subtraction.
- Add and subtract within 20.
- Work with equal groups of objects to gain foundations for multiplication.

Number and Operations in Base Ten

- Understand place value.
- Use place value understanding and properties of operations to add and subtract.

Measurement and Data

- Measure and estimate lengths in standard units.
- Relate addition and subtraction to length.
- Work with time and money.
- Represent and interpret data.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 2.OA

Represent and solve problems involving addition and subtraction.

1. Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]

Add and subtract within 20.

2. Fluently add and subtract within 20 using mental strategies.[2] By end of Grade 2, know from memory all sums of two one-digit numbers.

Work with equal groups of objects to gain foundations for multiplication.

3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.

4. Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten 2.NBT

Understand place value.

1. Understand that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones. Understand the following as special cases:

   a. 100 can be thought of as a bundle of ten tens — called a "hundred."

   b. The numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

2. Count within 1000; skip-count by 5s, 10s, and 100s.

3. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

4. Compare two three-digit numbers based on meanings of the hundreds, tens, and ones digits, using >, =, and < symbols to record the results of comparisons.

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

6. Add up to four two-digit numbers using strategies based on place value and properties of operations.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

8. Mentally add 10 or 100 to a given number 100–900, and mentally subtract 10 or 100 from a given number 100–900.

9. Explain why addition and subtraction strategies work, using place value and the properties of operations.[3]

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[1]See Glossary, Table 1.
[2]See standard 1.OA.6 for a list of mental strategies.
[3]Explanations may be supported by drawings or objects.

Measurement and Data 2.MD

Measure and estimate lengths in standard units.

1. Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
2. Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
3. Estimate lengths using units of inches, feet, centimeters, and meters.
4. Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Relate addition and subtraction to length.

5. Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.
6. Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

Work with time and money.

7. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
8. Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. *Example: If you have 2 dimes and 3 pennies, how many cents do you have?*

Represent and interpret data.

9. Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
10. Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems[4] using information presented in a bar graph.

Geometry 2.G

Reason with shapes and their attributes.

1. Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces.[5] Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
2. Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
3. Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words *halves, thirds, half of, a third of,* etc., and describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.

[4]See Glossary, Table 1.
[5]Sizes are compared directly or visually, not compared by measuring.

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- Represent and solve problems involving multiplication and division.
- Understand properties of multiplication and the relationship between multiplication and division.
- Multiply and divide within 100.
- Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Number and Operations in Base Ten

- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Develop understanding of fractions as numbers.

Measurement and Data

- Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.
- Represent and interpret data.
- Geometric measurement: understand concepts of area and relate area to multiplication and to addition.
- Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

Geometry

- Reason with shapes and their attributes.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = [] ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

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[1]See Glossary, Table 2.
[2]Students need not use formal terms for these properties.
[3]This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten — 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] — 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.
   b. Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data — 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

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[4] A range of algorithms may be used.

[5] Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.

   a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area, and can be used to measure area.

   b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.

6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).

7. Relate area to the operations of multiplication and addition.

   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.

   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.

   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

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[6]Excludes compound units such as cm^3 and finding the geometric volume of a container.

[7]Excludes multiplicative comparison problems (problems involving notions of "times as much"; see Glossary, Table 2).

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Mathematics | Grade 4

In Grade 4, instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication, and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry.

(1) Students generalize their understanding of place value to 1,000,000, understanding the relative sizes of numbers in each place. They apply their understanding of models for multiplication (equal-sized groups, arrays, area models), place value, and properties of operations, in particular the distributive property, as they develop, discuss, and use efficient, accurate, and generalizable methods to compute products of multi-digit whole numbers. Depending on the numbers and the context, they select and accurately apply appropriate methods to estimate or mentally calculate products. They develop fluency with efficient procedures for multiplying whole numbers; understand and explain why the procedures work based on place value and properties of operations; and use them to solve problems. Students apply their understanding of models for division, place value, properties of operations, and the relationship of division to multiplication as they develop, discuss, and use efficient, accurate, and generalizable procedures to find quotients involving multi-digit dividends. They select and accurately apply appropriate methods to estimate and mentally calculate quotients, and interpret remainders based upon the context.

(2) Students develop understanding of fraction equivalence and operations with fractions. They recognize that two different fractions can be equal (e.g., 15/9 = 5/3), and they develop methods for generating and recognizing equivalent fractions. Students extend previous understandings about how fractions are built from unit fractions, composing fractions from unit fractions, decomposing fractions into unit fractions, and using the meaning of fractions and the meaning of multiplication to multiply a fraction by a whole number.

(3) Students describe, analyze, compare, and classify two-dimensional shapes. Through building, drawing, and analyzing two-dimensional shapes, students deepen their understanding of properties of two-dimensional objects and the use of them to solve problems involving symmetry.

Grade 4 Overview

Operations and Algebraic Thinking

- Use the four operations with whole numbers to solve problems.
- Gain familiarity with factors and multiples.
- Generate and analyze patterns.

Number and Operations in Base Ten

- Generalize place value understanding for multi-digit whole numbers.
- Use place value understanding and properties of operations to perform multi-digit arithmetic.

Number and Operations—Fractions

- Extend understanding of fraction equivalence and ordering.
- Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
- Understand decimal notation for fractions, and compare decimal fractions.

Measurement and Data

- Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.
- Represent and interpret data.
- Geometric measurement: understand concepts of angle and measure angles.

Geometry

- Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 4.OA

Use the four operations with whole numbers to solve problems.

1. Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.
2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.[1]
3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Gain familiarity with factors and multiples.

4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Generate and analyze patterns.

5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. *For example, given the rule "Add 3" and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.*

Number and Operations in Base Ten[2] 4.NBT

Generalize place value understanding for multi-digit whole numbers.

1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. *For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.*
2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
3. Use place value understanding to round multi-digit whole numbers to any place.

Use place value understanding and properties of operations to perform multi-digit arithmetic.

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.
5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

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[1]See Glossary, Table 2.
[2]Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Number and Operations—Fractions[3] 4.NF

Extend understanding of fraction equivalence and ordering.

1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.
2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.

3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.
   a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
   b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. *Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.*
   c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
   d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.
4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.
   a. Understand a fraction a/b as a multiple of $1/b$. *For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4).*
   b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. *For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.)*
   c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. *For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?*

[3]Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.

Understand decimal notation for fractions, and compare decimal fractions.

5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.[4] *For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.*

6. Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.*

7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

Measurement and Data 4.MD

Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.

1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. *For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...*

2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.*

Represent and interpret data.

4. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. *For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.*

Geometric measurement: understand concepts of angle and measure angles.

5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

   a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a "one-degree angle," and can be used to measure angles.

   b. An angle that turns through *n* one-degree angles is said to have an angle measure of *n* degrees.

[4]Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.

6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.

7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

Geometry	4.G

Draw and identify lines and angles, and classify shapes by properties of their lines and angles.

1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.

2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.

3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.

Mathematics | Grade 5

In Grade 5, instructional time should focus on three critical areas: (1) developing fluency with addition and subtraction of fractions, and developing understanding of the multiplication of fractions and of division of fractions in limited cases (unit fractions divided by whole numbers and whole numbers divided by unit fractions); (2) extending division to 2-digit divisors, integrating decimal fractions into the place value system and developing understanding of operations with decimals to hundredths, and developing fluency with whole number and decimal operations; and (3) developing understanding of volume.

(1) Students apply their understanding of fractions and fraction models to represent the addition and subtraction of fractions with unlike denominators as equivalent calculations with like denominators. They develop fluency in calculating sums and differences of fractions, and make reasonable estimates of them. Students also use the meaning of fractions, of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for multiplying and dividing fractions make sense. (Note: this is limited to the case of dividing unit fractions by whole numbers and whole numbers by unit fractions.)

(2) Students develop understanding of why division procedures work based on the meaning of base-ten numerals and properties of operations. They finalize fluency with multi-digit addition, subtraction, multiplication, and division. They apply their understandings of models for decimals, decimal notation, and properties of operations to add and subtract decimals to hundredths. They develop fluency in these computations, and make reasonable estimates of their results. Students use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain why the procedures for multiplying and dividing finite decimals make sense. They compute products and quotients of decimals to hundredths efficiently and accurately.

(3) Students recognize volume as an attribute of three-dimensional space. They understand that volume can be measured by finding the total number of same-size units of volume required to fill the space without gaps or overlaps. They understand that a 1-unit by 1-unit by 1-unit cube is the standard unit for measuring volume. They select appropriate units, strategies, and tools for solving problems that involve estimating and measuring volume. They decompose three-dimensional shapes and find volumes of right rectangular prisms by viewing them as decomposed into layers of arrays of cubes. They measure necessary attributes of shapes in order to determine volumes to solve real world and mathematical problems.

Grade 5 Overview

Operations and Algebraic Thinking

- Write and interpret numerical expressions.
- Analyze patterns and relationships.

Number and Operations in Base Ten

- Understand the place value system.
- Perform operations with multi-digit whole numbers and with decimals to hundredths.

Number and Operations—Fractions

- Use equivalent fractions as a strategy to add and subtract fractions.
- Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

Measurement and Data

- Convert like measurement units within a given measurement system.
- Represent and interpret data.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

Geometry

- Graph points on the coordinate plane to solve real-world and mathematical problems.
- Classify two-dimensional figures into categories based on their properties.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 5.OA

Write and interpret numerical expressions.

1. Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.
2. Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. *For example, express the calculation "add 8 and 7, then multiply by 2" as 2 × (8 + 7). Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.*

Analyze patterns and relationships.

3. Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. *For example, given the rule "Add 3" and the starting number 0, and given the rule "Add 6" and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.*

Number and Operations in Base Ten 5.NBT

Understand the place value system.

1. Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.
2. Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.
3. Read, write, and compare decimals to thousandths.
   a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 × 100 + 4 × 10 + 7 × 1 + 3 × (1/10) + 9 × (1/100) + 2 × (1/1000).
   b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.
4. Use place value understanding to round decimals to any place.

Perform operations with multi-digit whole numbers and with decimals to hundredths.

5. Fluently multiply multi-digit whole numbers using the standard algorithm.
6. Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Number and Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)*

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

3. Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

   a. Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.)*

   b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

5. Interpret multiplication as scaling (resizing), by:

   a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

   b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

6. Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

7. Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]

   a. Interpret division of a unit fraction by a non-zero whole number,

[1]Students able to multiply fractions in general can develop strategies to divide fractions in general, by reasoning about the relationship between multiplication and division. But division of a fraction by a fraction is not a requirement at this grade.

and compute such quotients. *For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.*

b. Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.*

c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?*

Measurement and Data 5.MD

Convert like measurement units within a given measurement system.

1. Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.

Represent and interpret data.

2. Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. *For example, given different measurements of liquid in identical beakers, find the amount of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.*

Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.

3. Recognize volume as an attribute of solid figures and understand concepts of volume measurement.

   a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume.

   b. A solid figure which can be packed without gaps or overlaps using *n* unit cubes is said to have a volume of *n* cubic units.

4. Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.

5. Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume.

   a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication.

   b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

   c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.

Geometry 5.G

Graph points on the coordinate plane to solve real-world and mathematical problems.

1. Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., *x*-axis and *x*-coordinate, *y*-axis and *y*-coordinate).
2. Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Classify two-dimensional figures into categories based on their properties.

3. Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. *For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.*
4. Classify two-dimensional figures in a hierarchy based on properties.

Mathematics | Grade 6

In Grade 6, instructional time should focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting, and using expressions and equations; and (4) developing understanding of statistical thinking.

(1) Students use reasoning about multiplication and division to solve ratio and rate problems about quantities. By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative size of quantities, students connect their understanding of multiplication and division with ratios and rates. Thus students expand the scope of problems for which they can use multiplication and division to solve problems, and they connect ratios and fractions. Students solve a wide variety of problems involving ratios and rates.

(2) Students use the meaning of fractions, the meanings of multiplication and division, and the relationship between multiplication and division to understand and explain why the procedures for dividing fractions make sense. Students use these operations to solve problems. Students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. They reason about the order and absolute value of rational numbers and about the location of points in all four quadrants of the coordinate plane.

(3) Students understand the use of variables in mathematical expressions. They write expressions and equations that correspond to given situations, evaluate expressions, and use expressions and formulas to solve problems. Students understand that expressions in different forms can be equivalent, and they use the properties of operations to rewrite expressions in equivalent forms. Students know that the solutions of an equation are the values of the variables that make the equation true. Students use properties of operations and the idea of maintaining the equality of both sides of an equation to solve simple one-step equations. Students construct and analyze tables, such as tables of quantities that are in equivalent ratios, and they use equations (such as $3x = y$) to describe relationships between quantities.

(4) Building on and reinforcing their understanding of number, students begin to develop their ability to think statistically. Students recognize that a data distribution may not have a definite center and that different ways to measure center yield different values. The median measures center in the sense that it is roughly the middle value. The mean measures center in the sense that it is the value that each data point would take on if the total of the data values were redistributed equally, and also in the sense that it is a balance point. Students recognize that a measure of variability (interquartile range or mean absolute deviation) can also be useful for summarizing data because two very different sets of data can have the same mean and

median yet be distinguished by their variability. Students learn to describe and summarize numerical data sets, identifying clusters, peaks, gaps, and symmetry, considering the context in which the data were collected.

Students in Grade 6 also build on their work with area in elementary school by reasoning about relationships among shapes to determine area, surface area, and volume. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. Using these methods, students discuss, develop, and justify formulas for areas of triangles and parallelograms. Students find areas of polygons and surface areas of prisms and pyramids by decomposing them into pieces whose area they can determine. They reason about right rectangular prisms with fractional side lengths to extend formulas for the volume of a right rectangular prism to fractional side lengths. They prepare for work on scale drawings and constructions in Grade 7 by drawing polygons in the coordinate plane.

Grade 6 Overview

Ratios and Proportional Relationships

- Understand ratio concepts and use ratio reasoning to solve problems.

The Number System

- Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
- Compute fluently with multi-digit numbers and find common factors and multiples.
- Apply and extend previous understandings of numbers to the system of rational numbers.

Expressions and Equations

- Apply and extend previous understandings of arithmetic to algebraic expressions.
- Reason about and solve one-variable equations and inequalities.
- Represent and analyze quantitative relationships between dependent and independent variables.

Geometry

- Solve real-world and mathematical problems involving area, surface area, and volume.

Statistics and Probability

- Develop understanding of statistical variability.
- Summarize and describe distributions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships 6.RP

Understand ratio concepts and use ratio reasoning to solve problems.

1. Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*
2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]
3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
   a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*
   c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.
   d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System 6.NS

Apply and extend previous understandings of multiplication and division to divide fractions by fractions.

1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?*

Compute fluently with multi-digit numbers and find common factors and multiples.

2. Fluently divide multi-digit numbers using the standard algorithm.
3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. *For example, express 36 + 8 as 4 (9 + 2).*

[1]Expectations for unit rates in this grade are limited to non-complex fractions.

Apply and extend previous understandings of numbers to the system of rational numbers.

5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

   a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.

   b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.

   c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

7. Understand ordering and absolute value of rational numbers.

   a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. *For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.*

   b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. *For example, write $-3\,°C > -7\,°C$ to express the fact that $-3\,°C$ is warmer than $-7\,°C$.*

   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. *For example, for an account balance of -30 dollars, write $|-30| = 30$ to describe the size of the debt in dollars.*

   d. Distinguish comparisons of absolute value from statements about order. *For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.*

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations — 6.EE

Apply and extend previous understandings of arithmetic to algebraic expressions.

1. Write and evaluate numerical expressions involving whole-number exponents.

2. Write, read, and evaluate expressions in which letters stand for numbers.

   a. Write expressions that record operations with numbers and with letters standing for numbers. *For example, express the calculation "Subtract y from 5" as $5 - y$.*

b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. *For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.*

c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6 s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*

3. Apply the properties of operations to generate equivalent expressions. *For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.*

4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.

7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.

8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. *For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.*

Geometry 6.G

Solve real-world and mathematical problems involving area, surface area, and volume.

1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\,w\,h$ and $V = b\,h$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability 6.SP

Develop understanding of statistical variability.

1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. *For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.*
2. Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.
3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

Summarize and describe distributions.

4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.
5. Summarize numerical data sets in relation to their context, such as by:
   a. Reporting the number of observations.
   b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
   c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
   d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

Mathematics | Grade 7

In Grade 7, instructional time should focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions, and working with two- and three-dimensional shapes to solve problems involving area, surface area, and volume; and (4) drawing inferences about populations based on samples.

(1) Students extend their understanding of ratios and develop understanding of proportionality to solve single- and multi-step problems. Students use their understanding of ratios and proportionality to solve a wide variety of percent problems, including those involving discounts, interest, taxes, tips, and percent increase or decrease. Students solve problems about scale drawings by relating corresponding lengths between the objects or by using the fact that relationships of lengths within an object are preserved in similar objects. Students graph proportional relationships and understand the unit rate informally as a measure of the steepness of the related line, called the slope. They distinguish proportional relationships from other relationships.

(2) Students develop a unified understanding of number, recognizing fractions, decimals (that have a finite or a repeating decimal representation), and percents as different representations of rational numbers. Students extend addition, subtraction, multiplication, and division to all rational numbers, maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division. By applying these properties, and by viewing negative numbers in terms of everyday contexts (e.g., amounts owed or temperatures below zero), students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers. They use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.

(3) Students continue their work with area from Grade 6, solving problems involving the area and circumference of a circle and surface area of three-dimensional objects. In preparation for work on congruence and similarity in Grade 8 they reason about relationships among two-dimensional figures using scale drawings and informal geometric constructions, and they gain familiarity with the relationships between angles formed by intersecting lines. Students work with three-dimensional figures, relating them to two-dimensional figures by examining cross-sections. They solve real-world and mathematical problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes and right prisms.

(4) Students build on their previous work with single data distributions to compare two data distributions and address questions about differences between populations. They begin informal work with random sampling to generate data sets and learn about the importance of representative samples for drawing inferences.

Grade 7 Overview

Ratios and Proportional Relationships

- Analyze proportional relationships and use them to solve real-world and mathematical problems.

The Number System

- Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Expressions and Equations

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Geometry

- Draw, construct and describe geometrical figures and describe the relationships between them.
- Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

Statistics and Probability

- Use random sampling to draw inferences about a population.
- Draw informal comparative inferences about two populations.
- Investigate chance processes and develop, use, and evaluate probability models.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Ratios and Proportional Relationships 7.RP

Analyze proportional relationships and use them to solve real-world and mathematical problems.

1. Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. *For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction $^{1/2}/_{1/4}$ miles per hour, equivalently 2 miles per hour.*
2. Recognize and represent proportional relationships between quantities.
   a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
   b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
   c. Represent proportional relationships by equations. *For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as $t = pn$.*
   d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and $(1, r)$ where r is the unit rate.
3. Use proportional relationships to solve multistep ratio and percent problems. *Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.*

The Number System 7.NS

Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

1. Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
   a. Describe situations in which opposite quantities combine to make 0. *For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.*
   b. Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
   c. Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
   d. Apply properties of operations as strategies to add and subtract rational numbers.
2. Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
   a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

c. Apply properties of operations as strategies to multiply and divide rational numbers.

d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

3. Solve real-world and mathematical problems involving the four operations with rational numbers.[1]

Expressions and Equations 7.EE

Use properties of operations to generate equivalent expressions.

1. Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

2. Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. *For example, $a + 0.05a = 1.05a$ means that "increase by 5%" is the same as "multiply by 1.05."*

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

3. Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*

4. Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

   a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. *For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?*

   b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. *For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.*

Geometry 7.G

Draw, construct, and describe geometrical figures and describe the relationships between them.

1. Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

[1]Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

2. Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

3. Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

4. Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

5. Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

6. Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability 7.SP

Use random sampling to draw inferences about a population.

1. Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

2. Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. *For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.*

Draw informal comparative inferences about two populations.

3. Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. *For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.*

4. Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. *For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.*

Investigate chance processes and develop, use, and evaluate probability models.

5. Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

6. Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. *For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.*

7. Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

   a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. *For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.*

   b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. *For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?*

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

   a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

   b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

   c. Design and use a simulation to generate frequencies for compound events. *For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?*

Mathematics | Grade 8

In Grade 8, instructional time should focus on three critical areas: (1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity, and congruence, and understanding and applying the Pythagorean Theorem.

(1) Students use linear equations and systems of linear equations to represent, analyze, and solve a variety of problems. Students recognize equations for proportions ($y/x = m$ or $y = mx$) as special linear equations ($y = mx + b$), understanding that the constant of proportionality (m) is the slope, and the graphs are lines through the origin. They understand that the slope (m) of a line is a constant rate of change, so that if the input or x-coordinate changes by an amount A, the output or y-coordinate changes by the amount $m \cdot A$. Students also use a linear equation to describe the association between two quantities in bivariate data (such as arm span vs. height for students in a classroom). At this grade, fitting the model, and assessing its fit to the data are done informally. Interpreting the model in the context of the data requires students to express a relationship between the two quantities in question and to interpret components of the relationship (such as slope and y-intercept) in terms of the situation.

Students strategically choose and efficiently implement procedures to solve linear equations in one variable, understanding that when they use the properties of equality and the concept of logical equivalence, they maintain the solutions of the original equation. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines in the plane; these intersect, are parallel, or are the same line. Students use linear equations, systems of linear equations, linear functions, and their understanding of slope of a line to analyze situations and solve problems.

(2) Students grasp the concept of a function as a rule that assigns to each input exactly one output. They understand that functions describe situations where one quantity determines another. They can translate among representations and partial representations of functions (noting that tabular and graphical representations may be partial representations), and they describe how aspects of the function are reflected in the different representations.

(3) Students use ideas about distance and angles, how they behave under translations, rotations, reflections, and dilations, and ideas about congruence and similarity to describe and analyze two-dimensional figures and to solve problems. Students show that the sum of the angles in a triangle is the angle formed by a straight line, and that various configurations of lines give rise to similar triangles because of the angles created when a transversal cuts parallel lines. Students understand the statement of the Pythagorean Theorem and its converse, and can explain why the Pythagorean Theorem holds, for example, by decomposing a square in two different ways. They apply the Pythagorean Theorem to find distances between points on the coordinate plane, to find lengths, and to analyze polygons. Students complete their work on volume by solving problems involving cones, cylinders, and spheres.

Grade 8 Overview

The Number System

- Know that there are numbers that are not rational, and approximate them by rational numbers.

Expressions and Equations

- Work with radicals and integer exponents.
- Understand the connections between proportional relationships, lines, and linear equations.
- Analyze and solve linear equations and pairs of simultaneous linear equations.

Functions

- Define, evaluate, and compare functions.
- Use functions to model relationships between quantities.

Geometry

- Understand congruence and similarity using physical models, transparencies, or geometry software.
- Understand and apply the Pythagorean Theorem.
- Solve real-world and mathematical problems involving volume of cylinders, cones and spheres.

Statistics and Probability

- Investigate patterns of association in bivariate data.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Number System 8.NS

Know that there are numbers that are not rational, and approximate them by rational numbers.

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

Expressions and Equations 8.EE

Work with radicals and integer exponents.

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions. *For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.*

2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. *For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.*

4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. *For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.*

6. Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.

   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers).

   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.

   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

   c. Solve real-world and mathematical problems leading to two linear equations in two variables. *For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.*

Functions 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.[1]

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.*

3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.*

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry 8.G

Understand congruence and similarity using physical models, transparencies, or geometry software.

1. Verify experimentally the properties of rotations, reflections, and translations:

   a. Lines are taken to lines, and line segments to line segments of the same length.

   b. Angles are taken to angles of the same measure.

   c. Parallel lines are taken to parallel lines.

2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

[1]Function notation is not required in Grade 8.

3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*

Understand and apply the Pythagorean Theorem.

6. Explain a proof of the Pythagorean Theorem and its converse.

7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability 8.SP

Investigate patterns of association in bivariate data.

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. *For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?*

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (*). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School—Number and Quantity

Numbers and Number Systems. During the years from kindergarten to eighth grade, students must repeatedly extend their conception of number. At first, "number" means "counting number": 1, 2, 3... Soon after that, 0 is used to represent "none" and the whole numbers are formed by the counting numbers together with zero. The next extension is fractions. At first, fractions are barely numbers and tied strongly to pictorial representations. Yet by the time students understand division of fractions, they have a strong concept of fractions as numbers and have connected them, via their decimal representations, with the base-ten system used to represent the whole numbers. During middle school, fractions are augmented by negative fractions to form the rational numbers. In Grade 8, students extend this system once more, augmenting the rational numbers with the irrational numbers to form the real numbers. In high school, students will be exposed to yet another extension of number, when the real numbers are augmented by the imaginary numbers to form the complex numbers.

With each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, real numbers, and complex numbers—the four operations stay the same in two important ways: They have the commutative, associative, and distributive properties and their new meanings are consistent with their previous meanings.

Extending the properties of whole-number exponents leads to new and productive notation. For example, properties of whole-number exponents suggest that $(5^{1/3})^3$ should be $5^{(1/3)3} = 5^1 = 5$ and that $5^{1/3}$ should be the cube root of 5.

Calculators, spreadsheets, and computer algebra systems can provide ways for students to become better acquainted with these new number systems and their notation. They can be used to generate data for numerical experiments, to help understand the workings of matrix, vector, and complex number algebra, and to experiment with non-integer exponents.

Quantities. In real world problems, the answers are usually not numbers but quantities: numbers with units, which involves measurement. In their work in measurement up through Grade 8, students primarily measure commonly used attributes such as length, area, and volume. In high school, students encounter a wider variety of units in modeling, e.g., acceleration, currency conversions, derived quantities such as person-hours and heating degree days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages. They also encounter novel situations in which they themselves must conceive the attributes of interest. For example, to find a good measure of overall highway safety, they might propose measures such as fatalities per year, fatalities per year per driver, or fatalities per vehicle-mile traveled. Such a conceptual process is sometimes called quantification. Quantification is important for science, as when surface area suddenly "stands out" as an important variable in evaporation. Quantification is also important for companies, which must conceptualize relevant attributes and create or choose suitable measures for them.

Number and Quantity Overview

The Real Number System

- Extend the properties of exponents to rational exponents
- Use properties of rational and irrational numbers.

Quantities

- Reason quantitatively and use units to solve problems

The Complex Number System

- Perform arithmetic operations with complex numbers
- Represent complex numbers and their operations on the complex plane
- Use complex numbers in polynomial identities and equations

Vector and Matrix Quantities

- Represent and model with vector quantities.
- Perform operations on vectors.
- Perform operations on matrices and use matrices in applications.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Real Number System — N-RN

Extend the properties of exponents to rational exponents.

1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define* $5^{1/3}$ *to be the cube root of 5 because we want* $(5^{1/3})^3 = 5^{(1/3)3}$ *to hold, so* $(5^{1/3})^3$ *must equal 5.*
2. Rewrite expressions involving radicals and rational exponents using the properties of exponents.

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities* — N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

The Complex Number System — N-CN

Perform arithmetic operations with complex numbers.

1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.
2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
3. (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Represent complex numbers and their operations on the complex plane.

4. (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
5. (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus* 2 *and argument* 120°.
6. (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.

Use complex numbers in polynomial identities and equations.

7. Solve quadratic equations with real coefficients that have complex solutions.
8. (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x - 2i)$.
9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Vector and Matrix Quantities N-VM

Represent and model with vector quantities.

1. (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).
2. (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
3. (+) Solve problems involving velocity and other quantities that can be represented by vectors.

Perform operations on vectors.

4. (+) Add and subtract vectors.
   a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
   b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
   c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
5. (+) Multiply a vector by a scalar.
   a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.
   b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Perform operations on matrices and use matrices in applications.

6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
7. (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
8. (+) Add, subtract, and multiply matrices of appropriate dimensions.
9. (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
10. (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
11. (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
12. (+) Work with 2×2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.

Mathematics | High School—Algebra

Expressions. An expression is a record of a computation with numbers, symbols that represent numbers, arithmetic operations, exponentiation, and, at more advanced levels, the operation of evaluating a function. Conventions about the use of parentheses and the order of operations assure that each expression is unambiguous. Creating an expression that describes a computation involving a general quantity requires the ability to express the computation in general terms, abstracting from specific instances.

Reading an expression with comprehension involves analysis of its underlying structure. This may suggest a different but equivalent way of writing the expression that exhibits some different aspect of its meaning. For example, $p + 0.05p$ can be interpreted as the addition of a 5% tax to a price p. Rewriting $p + 0.05p$ as $1.05p$ shows that adding a tax is the same as multiplying the price by a constant factor.

Algebraic manipulations are governed by the properties of operations and exponents, and the conventions of algebraic notation. At times, an expression is the result of applying operations to simpler expressions. For example, $p + 0.05p$ is the sum of the simpler expressions p and $0.05p$. Viewing an expression as the result of operation on simpler expressions can sometimes clarify its underlying structure.

A spreadsheet or a computer algebra system (CAS) can be used to experiment with algebraic expressions, perform complicated algebraic manipulations, and understand how algebraic manipulations behave.

Equations and inequalities. An equation is a statement of equality between two expressions, often viewed as a question asking for which values of the variables the expressions on either side are in fact equal. These values are the solutions to the equation. An identity, in contrast, is true for all values of the variables; identities are often developed by rewriting an expression in an equivalent form.

The solutions of an equation in one variable form a set of numbers; the solutions of an equation in two variables form a set of ordered pairs of numbers, which can be plotted in the coordinate plane. Two or more equations and/or inequalities form a system. A solution for such a system must satisfy every equation and inequality in the system.

An equation can often be solved by successively deducing from it one or more simpler equations. For example, one can add the same constant to both sides without changing the solutions, but squaring both sides might lead to extraneous solutions. Strategic competence in solving includes looking ahead for productive manipulations and anticipating the nature and number of solutions.

Some equations have no solutions in a given number system, but have a solution in a larger system. For example, the solution of $x + 1 = 0$ is an integer, not a whole number; the solution of $2x + 1 = 0$ is a rational number, not an integer; the solutions of $x^2 - 2 = 0$ are real numbers, not rational numbers; and the solutions of $x^2 + 2 = 0$ are complex numbers, not real numbers.

The same solution techniques used to solve equations can be used to rearrange formulas. For example, the formula for the area of a trapezoid, $A = ((b_1+b_2)/2)h$, can be solved for h using the same deductive process.

Inequalities can be solved by reasoning about the properties of inequality. Many, but not all, of the properties of equality continue to hold for inequalities and can be useful in solving them.

Connections to Functions and Modeling. Expressions can define functions, and equivalent expressions define the same function. Asking when two functions have the same value for the same input leads to an equation; graphing the two functions allows for finding approximate solutions of the equation. Converting a verbal description to an equation, inequality, or system of these is an essential skill in modeling.

Algebra Overview

Seeing Structure in Expressions

- Interpret the structure of expressions
- Write expressions in equivalent forms to solve problems

Arithmetic with Polynomials and Rational Expressions

- Perform arithmetic operations on polynomials
- Understand the relationship between zeros and factors of polynomials
- Use polynomial identities to solve problems
- Rewrite rational expressions

Creating Equations

- Create equations that describe numbers or relationships

Reasoning with Equations and Inequalities

- Understand solving equations as a process of reasoning and explain the reasoning
- Solve equations and inequalities in one variable
- Solve systems of equations
- Represent and solve equations and inequalities graphically

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Seeing Structure in Expressions — A-SSE

Interpret the structure of expressions

1. Interpret expressions that represent a quantity in terms of its context.*
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret* $P(1+r)^n$ *as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
   a. Factor a quadratic expression to reveal the zeros of the function it defines.
   b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
   c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*
4. Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments.**

Arithmetic with Polynomials and Rational Expressions — A-APR

Perform arithmetic operations on polynomials

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials

2. Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.
3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Use polynomial identities to solve problems

4. Prove polynomial identities and use them to describe numerical relationships. *For example, the polynomial identity* $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ *can be used to generate Pythagorean triples.*
5. (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[1]

[1]The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.

Rewrite rational expressions

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Creating Equations* — A-CED

Create equations that describe numbers or relationships

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*

4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law $V = IR$ to highlight resistance R.*

Reasoning with Equations and Inequalities — A-REI

Understand solving equations as a process of reasoning and explain the reasoning

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

4. Solve quadratic equations in one variable.

   a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

   b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.

6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line* $y = -3x$ and the circle $x^2 + y^2 = 3$.

8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.

9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).

Represent and solve equations and inequalities graphically

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*

12. Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Overview

Interpreting Functions

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations

Building Functions

- Build a function that models a relationship between two quantities
- Build new functions from existing functions

Linear, Quadratic, and Exponential Models

- Construct and compare linear, quadratic, and exponential models and solve problems
- Interpret expressions for functions in terms of the situation they model

Trigonometric Functions

- Extend the domain of trigonometric functions using the unit circle
- Model periodic phenomena with trigonometric functions
- Prove and apply trigonometric identities

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Functions F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.**

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function* $h(n)$ *gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*

   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.

   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x-1)$ *for* $x \neq 1$.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models* F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

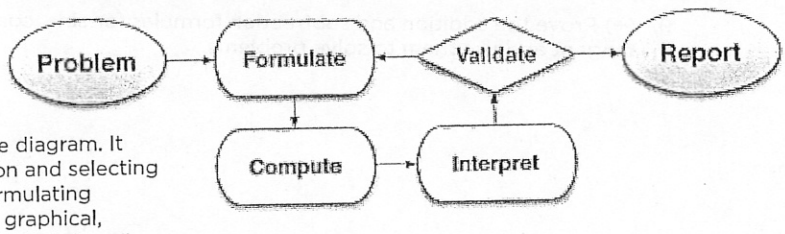
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it

is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol* (*).

Mathematics | High School—Geometry

An understanding of the attributes and relationships of geometric objects can be applied in diverse contexts—interpreting a schematic drawing, estimating the amount of wood needed to frame a sloping roof, rendering computer graphics, or designing a sewing pattern for the most efficient use of material.

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). Euclidean geometry is characterized most importantly by the Parallel Postulate, that through a point not on a given line there is exactly one parallel line. (Spherical geometry, in contrast, has no parallel lines.)

During high school, students begin to formalize their geometry experiences from elementary and middle school, using more precise definitions and developing careful proofs. Later in college some students develop Euclidean and other geometries carefully from a small set of axioms.

The concepts of congruence, similarity, and symmetry can be understood from the perspective of geometric transformation. Fundamental are the rigid motions: translations, rotations, reflections, and combinations of these, all of which are here assumed to preserve distance and angles (and therefore shapes generally). Reflections and rotations each explain a particular type of symmetry, and the symmetries of an object offer insight into its attributes—as when the reflective symmetry of an isosceles triangle assures that its base angles are congruent.

In the approach taken here, two geometric figures are defined to be congruent if there is a sequence of rigid motions that carries one onto the other. This is the principle of superposition. For triangles, congruence means the equality of all corresponding pairs of sides and all corresponding pairs of angles. During the middle grades, through experiences drawing triangles from given conditions, students notice ways to specify enough measures in a triangle to ensure that all triangles drawn with those measures are congruent. Once these triangle congruence criteria (ASA, SAS, and SSS) are established using rigid motions, they can be used to prove theorems about triangles, quadrilaterals, and other geometric figures.

Similarity transformations (rigid motions followed by dilations) define similarity in the same way that rigid motions define congruence, thereby formalizing the similarity ideas of "same shape" and "scale factor" developed in the middle grades. These transformations lead to the criterion for triangle similarity that two pairs of corresponding angles are congruent.

The definitions of sine, cosine, and tangent for acute angles are founded on right triangles and similarity, and, with the Pythagorean Theorem, are fundamental in many real-world and theoretical situations. The Pythagorean Theorem is generalized to non-right triangles by the Law of Cosines. Together, the Laws of Sines and Cosines embody the triangle congruence criteria for the cases where three pieces of information suffice to completely solve a triangle. Furthermore, these laws yield two possible solutions in the ambiguous case, illustrating that Side-Side-Angle is not a congruence criterion.

Analytic geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving. Just as the number line associates numbers with locations in one dimension, a pair of perpendicular axes associates pairs of numbers with locations in two dimensions. This correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof. Geometric transformations of the graphs of equations correspond to algebraic changes in their equations.

Dynamic geometry environments provide students with experimental and modeling tools that allow them to investigate geometric phenomena in much the same way as computer algebra systems allow them to experiment with algebraic phenomena.

Connections to Equations. The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa. The solution set of an equation becomes a geometric curve, making visualization a tool for doing and understanding algebra. Geometric shapes can be described by equations, making algebraic manipulation into a tool for geometric understanding, modeling, and proof.

Geometry Overview

Congruence

- Experiment with transformations in the plane
- Understand congruence in terms of rigid motions
- Prove geometric theorems
- Make geometric constructions

Similarity, Right Triangles, and Trigonometry

- Understand similarity in terms of similarity transformations
- Prove theorems involving similarity
- Define trigonometric ratios and solve problems involving right triangles
- Apply trigonometry to general triangles

Circles

- Understand and apply theorems about circles
- Find arc lengths and areas of sectors of circles

Expressing Geometric Properties with Equations

- Translate between the geometric description and the equation for a conic section
- Use coordinates to prove simple geometric theorems algebraically

Geometric Measurement and Dimension

- Explain volume formulas and use them to solve problems
- Visualize relationships between two-dimensional and three-dimensional objects

Modeling with Geometry

- Apply geometric concepts in modeling situations

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Congruence G-CO

Experiment with transformations in the plane

1. Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
2. Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
3. Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
4. Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
5. Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

Understand congruence in terms of rigid motions

6. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
7. Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
8. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.

Prove geometric theorems

9. Prove theorems about lines and angles. *Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.*
10. Prove theorems about triangles. *Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.*
11. Prove theorems about parallelograms. *Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.*

Make geometric constructions

12. Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.*
13. Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

Similarity, Right Triangles, and Trigonometry G-SRT

Understand similarity in terms of similarity transformations

1. Verify experimentally the properties of dilations given by a center and a scale factor:
   a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
   b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
2. Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
3. Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Prove theorems involving similarity

4. Prove theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*
5. Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Define trigonometric ratios and solve problems involving right triangles

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

Apply trigonometry to general triangles

9. (+) Derive the formula $A = 1/2\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.
11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Circles G-C

Understand and apply theorems about circles

1. Prove that all circles are similar.
2. Identify and describe relationships among inscribed angles, radii, and chords. *Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.*
3. Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
4. (+) Construct a tangent line from a point outside a given circle to the circle.

Find arc lengths and areas of sectors of circles

5. Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.

Expressing Geometric Properties with Equations — G-GPE

Translate between the geometric description and the equation for a conic section

1. Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
2. Derive the equation of a parabola given a focus and directrix.
3. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

Use coordinates to prove simple geometric theorems algebraically

4. Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).*
5. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
6. Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
7. Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

Geometric Measurement and Dimension — G-GMD

Explain volume formulas and use them to solve problems

1. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments.*
2. (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
3. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Visualize relationships between two-dimensional and three-dimensional objects

4. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Modeling with Geometry — G-MG

Apply geometric concepts in modeling situations

1. Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*
2. Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*
3. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).*

Mathematics | High School—Statistics and Probability★

Decisions or predictions are often based on data—numbers in context. These decisions or predictions would be easy if the data always sent a clear message, but the message is often obscured by variability. Statistics provides tools for describing variability in data and for making informed decisions that take it into account.

Data are gathered, displayed, summarized, examined, and interpreted to discover patterns and deviations from patterns. Quantitative data can be described in terms of key characteristics: measures of shape, center, and spread. The shape of a data distribution might be described as symmetric, skewed, flat, or bell shaped, and it might be summarized by a statistic measuring center (such as mean or median) and a statistic measuring spread (such as standard deviation or interquartile range). Different distributions can be compared numerically using these statistics or compared visually using plots. Knowledge of center and spread are not enough to describe a distribution. Which statistics to compare, which plots to use, and what the results of a comparison might mean, depend on the question to be investigated and the real-life actions to be taken.

Randomization has two important uses in drawing statistical conclusions. First, collecting data from a random sample of a population makes it possible to draw valid conclusions about the whole population, taking variability into account. Second, randomly assigning individuals to different treatments allows a fair comparison of the effectiveness of those treatments. A statistically significant outcome is one that is unlikely to be due to chance alone, and this can be evaluated only under the condition of randomness. The conditions under which data are collected are important in drawing conclusions from the data; in critically reviewing uses of statistics in public media and other reports, it is important to consider the study design, how the data were gathered, and the analyses employed as well as the data summaries and the conclusions drawn.

Random processes can be described mathematically by using a probability model: a list or description of the possible outcomes (the sample space), each of which is assigned a probability. In situations such as flipping a coin, rolling a number cube, or drawing a card, it might be reasonable to assume various outcomes are equally likely. In a probability model, sample points represent outcomes and combine to make up events; probabilities of events can be computed by applying the Addition and Multiplication Rules. Interpreting these probabilities relies on an understanding of independence and conditional probability, which can be approached through the analysis of two-way tables.

Technology plays an important role in statistics and probability by making it possible to generate plots, regression functions, and correlation coefficients, and to simulate many possible outcomes in a short amount of time.

Connections to Functions and Modeling. Functions may be used to describe data; if the data suggest a linear relationship, the relationship can be modeled with a regression line, and its strength and direction can be expressed through a correlation coefficient.

Statistics and Probability Overview

Interpreting Categorical and Quantitative Data

- Summarize, represent, and interpret data on a single count or measurement variable
- Summarize, represent, and interpret data on two categorical and quantitative variables
- Interpret linear models

Making Inferences and Justifying Conclusions

- Understand and evaluate random processes underlying statistical experiments
- Make inferences and justify conclusions from sample surveys, experiments and observational studies

Conditional Probability and the Rules of Probability

- Understand independence and conditional probability and use them to interpret data
- Use the rules of probability to compute probabilities of compound events in a uniform probability model

Using Probability to Make Decisions

- Calculate expected values and use them to solve problems
- Use probability to evaluate outcomes of decisions

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
   a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
   b. Informally assess the fit of a function by plotting and analyzing residuals.
   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.

4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P(A* and *B)/P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model

6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.
7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

Using Probability to Make Decisions S-MD

Calculate expected values and use them to solve problems

1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.

3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*

4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

Use probability to evaluate outcomes of decisions

5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.

   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*

   b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).

7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Note on courses and transitions

The high school portion of the Standards for Mathematical Content specifies the mathematics all students should study for college and career readiness. These standards do not mandate the sequence of high school courses. However, the organization of high school courses is a critical component to implementation of the standards. To that end, sample high school pathways for mathematics – in both a traditional course sequence (Algebra I, Geometry, and Algebra II) as well as an integrated course sequence (Mathematics 1, Mathematics 2, Mathematics 3) – will be made available shortly after the release of the final Common Core State Standards. It is expected that additional model pathways based on these standards will become available as well.

The standards themselves do not dictate curriculum, pedagogy, or delivery of content. In particular, states may handle the transition to high school in different ways. For example, many students in the U.S. today take Algebra I in the 8th grade, and in some states this is a requirement. The K-7 standards contain the prerequisites to prepare students for Algebra I by 8th grade, and the standards are designed to permit states to continue existing policies concerning Algebra I in 8th grade.

A second major transition is the transition from high school to post-secondary education for college and careers. The evidence concerning college and career readiness shows clearly that the knowledge, skills, and practices important for readiness include a great deal of mathematics prior to the boundary defined by (+) symbols in these standards. Indeed, some of the highest priority content for college and career readiness comes from Grades 6-8. This body of material includes powerfully useful proficiencies such as applying ratio reasoning in real-world and mathematical problems, computing fluently with positive and negative fractions and decimals, and solving real-world and mathematical problems involving angle measure, area, surface area, and volume. Because important standards for college and career readiness are distributed across grades and courses, systems for evaluating college and career readiness should reach as far back in the standards as Grades 6-8. It is important to note as well that cut scores or other information generated by assessment systems for college and career readiness should be developed in collaboration with representatives from higher education and workforce development programs, and should be validated by subsequent performance of students in college and the workforce.

Glossary

Addition and subtraction within 5, 10, 20, 100, or 1000. Addition or subtraction of two whole numbers with whole number answers, and with sum or minuend in the range 0-5, 0-10, 0-20, or 0-100, respectively. Example: 8 + 2 = 10 is an addition within 10, 14 – 5 = 9 is a subtraction within 20, and 55 – 18 = 37 is a subtraction within 100.

Additive inverses. Two numbers whose sum is 0 are additive inverses of one another. Example: $^3/_4$ and $-^3/_4$ are additive inverses of one another because $^3/_4 + (-^3/_4) = (-^3/_4) + ^3/_4 = 0$.

Associative property of addition. See Table 3 in this Glossary.

Associative property of multiplication. See Table 3 in this Glossary.

Bivariate data. Pairs of linked numerical observations. Example: a list of heights and weights for each player on a football team.

Box plot. A method of visually displaying a distribution of data values by using the median, quartiles, and extremes of the data set. A box shows the middle 50% of the data.[1]

Commutative property. See Table 3 in this Glossary.

Complex fraction. A fraction $^A/_B$ where A and/or B are fractions (B nonzero).

Computation algorithm. A set of predefined steps applicable to a class of problems that gives the correct result in every case when the steps are carried out correctly. *See also:* computation strategy.

Computation strategy. Purposeful manipulations that may be chosen for specific problems, may not have a fixed order, and may be aimed at converting one problem into another. *See also:* computation algorithm.

Congruent. Two plane or solid figures are congruent if one can be obtained from the other by rigid motion (a sequence of rotations, reflections, and translations).

Counting on. A strategy for finding the number of objects in a group without having to count every member of the group. For example, if a stack of books is known to have 8 books and 3 more books are added to the top, it is not necessary to count the stack all over again. One can find the total by *counting on*—pointing to the top book and saying "eight," following this with "nine, ten, eleven. There are eleven books now."

Dot plot. *See:* line plot.

Dilation. A transformation that moves each point along the ray through the point emanating from a fixed center, and multiplies distances from the center by a common scale factor.

Expanded form. A multi-digit number is expressed in expanded form when it is written as a sum of single-digit multiples of powers of ten. For example, 643 = 600 + 40 + 3.

Expected value. For a random variable, the weighted average of its possible values, with weights given by their respective probabilities.

First quartile. For a data set with median M, the first quartile is the median of the data values less than M. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the first quartile is 6.[2] *See also:* median, third quartile, interquartile range.

Fraction. A number expressible in the form $^a/_b$ where a is a whole number and b is a positive whole number. (The word *fraction* in these standards always refers to a non-negative number.) *See also:* rational number.

Identity property of 0. See Table 3 in this Glossary.

Independently combined probability models. Two probability models are said to be combined independently if the probability of each ordered pair in the combined model equals the product of the original probabilities of the two individual outcomes in the ordered pair.

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[1]Adapted from Wisconsin Department of Public Instruction, http://dpi.wi.gov/standards/mathglos.html, accessed March 2, 2010.

[2]Many different methods for computing quartiles are in use. The method defined here is sometimes called the Moore and McCabe method. See Langford, E., "Quartiles in Elementary Statistics," *Journal of Statistics Education* Volume 14, Number 3 (2006).

Integer. A number expressible in the form *a* or –*a* for some whole number *a*.

Interquartile Range. A measure of variation in a set of numerical data, the interquartile range is the distance between the first and third quartiles of the data set. Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the interquartile range is 15 – 6 = 9. *See also:* first quartile, third quartile.

Line plot. A method of visually displaying a distribution of data values where each data value is shown as a dot or mark above a number line. Also known as a dot plot.[3]

Mean. A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.[4] Example: For the data set {1, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean is 21.

Mean absolute deviation. A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the mean absolute deviation is 20.

Median. A measure of center in a set of numerical data. The median of a list of values is the value appearing at the center of a sorted version of the list—or the mean of the two central values, if the list contains an even number of values. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 90}, the median is 11.

Midline. In the graph of a trigonometric function, the horizontal line halfway between its maximum and minimum values.

Multiplication and division within 100. Multiplication or division of two whole numbers with whole number answers, and with product or dividend in the range 0-100. Example: 72 ÷ 8 = 9.

Multiplicative inverses. Two numbers whose product is 1 are multiplicative inverses of one another. Example: 3/4 and 4/3 are multiplicative inverses of one another because 3/4 × 4/3 = 4/3 × 3/4 = 1.

Number line diagram. A diagram of the number line used to represent numbers and support reasoning about them. In a number line diagram for measurement quantities, the interval from 0 to 1 on the diagram represents the unit of measure for the quantity.

Percent rate of change. A rate of change expressed as a percent. Example: if a population grows from 50 to 55 in a year, it grows by 5/50 = 10% per year.

Probability distribution. The set of possible values of a random variable with a probability assigned to each.

Properties of operations. See Table 3 in this Glossary.

Properties of equality. See Table 4 in this Glossary.

Properties of inequality. See Table 5 in this Glossary.

Properties of operations. See Table 3 in this Glossary.

Probability. A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (such as tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).

Probability model. A probability model is used to assign probabilities to outcomes of a chance process by examining the nature of the process. The set of all outcomes is called the sample space, and their probabilities sum to 1. *See also:* uniform probability model.

Random variable. An assignment of a numerical value to each outcome in a sample space.

Rational expression. A quotient of two polynomials with a non-zero denominator.

Rational number. A number expressible in the form a/b or $-a/b$ for some fraction a/b. The rational numbers include the integers.

Rectilinear figure. A polygon all angles of which are right angles.

Rigid motion. A transformation of points in space consisting of a sequence of

[3]Adapted from Wisconsin Department of Public Instruction, *op. cit.*
[4]To be more precise, this defines the *arithmetic mean*.

one or more translations, reflections, and/or rotations. Rigid motions are here assumed to preserve distances and angle measures.

Repeating decimal. The decimal form of a rational number. *See also:* terminating decimal.

Sample space. In a probability model for a random process, a list of the individual outcomes that are to be considered.

Scatter plot. A graph in the coordinate plane representing a set of bivariate data. For example, the heights and weights of a group of people could be displayed on a scatter plot.[5]

Similarity transformation. A rigid motion followed by a dilation.

Tape diagram. A drawing that looks like a segment of tape, used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.

Terminating decimal. A decimal is called terminating if its repeating digit is 0.

Third quartile. For a data set with median *M*, the third quartile is the median of the data values greater than *M*. Example: For the data set {2, 3, 6, 7, 10, 12, 14, 15, 22, 120}, the third quartile is 15. *See also:* median, first quartile, interquartile range.

Transitivity principle for indirect measurement. If the length of object A is greater than the length of object B, and the length of object B is greater than the length of object C, then the length of object A is greater than the length of object C. This principle applies to measurement of other quantities as well.

Uniform probability model. A probability model which assigns equal probability to all outcomes. *See also:* probability model.

Vector. A quantity with magnitude and direction in the plane or in space, defined by an ordered pair or triple of real numbers.

Visual fraction model. A tape diagram, number line diagram, or area model.

Whole numbers. The numbers 0, 1, 2, 3, ...

[5]Adapted from Wisconsin Department of Public Instruction, *op. cit.*

TABLE 1. Common addition and subtraction situations.[6]

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? 2 + 3 = ?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? 2 + ? = 5	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? ? + 3 = 5
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? 5 – 2 = ?	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? 5 – ? = 3	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? ? – 2 = 3

	Total Unknown	Addend Unknown	Both Addends Unknown[1]
Put Together/ Take Apart[2]	Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?	Five apples are on the table. Three are red and the rest are green. How many apples are green? 3 + ? = 5, 5 – 3 = ?	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? 5 = 0 + 5, 5 = 5 + 0 5 = 1 + 4, 5 = 4 + 1 5 = 2 + 3, 5 = 3 + 2

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare[3]	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 – 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = ?, 3 + 2 = ?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? 5 – 3 = ?, ? + 3 = 5

[1]These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.
[2]Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.
[3]For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

[6]Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common multiplication and division situations.[7]

	Unknown Product $3 \times 6 = ?$	Group Size Unknown ("How many in each group?" Division) $3 \times ? = 18$, and $18 \div 3 = ?$	Number of Groups Unknown ("How many groups?" Division) $? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? *Measurement example*. You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? *Measurement example*. You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? *Measurement example*. You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,[4] Area[5]	There are 3 rows of apples with 6 apples in each row. How many apples are there? *Area example*. What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? *Area example*. A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? *Area example*. A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs $6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? *Measurement example*. A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs $18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? *Measurement example*. A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs $18 and a blue hat costs $6. How many times as much does the red hat cost as the blue hat? *Measurement example*. A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

[4]The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

[5]Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

[7]The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

TABLE 3. The properties of operations. Here a, b and c stand for arbitrary numbers in a given number system. The properties of operations apply to the rational number system, the real number system, and the complex number system.

Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a there exists $-a$ so that $a + (-a) = (-a) + a = 0$.
Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$
Commutative property of multiplication	$a \times b = b \times a$
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$.
Distributive property of multiplication over addition	$a \times (b + c) = a \times b + a \times c$

TABLE 4. The properties of equality. Here a, b and c stand for arbitrary numbers in the rational, real, or complex number systems.

Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$ and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \times c = b \times c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

TABLE 5. The properties of inequality. Here a, b and c stand for arbitrary numbers in the rational or real number systems.

Exactly one of the following is true: $a < b$, $a = b$, $a > b$.

If $a > b$ and $b > c$ then $a > c$.

If $a > b$, then $b < a$.

If $a > b$, then $-a < -b$.

If $a > b$, then $a \pm c > b \pm c$.

If $a > b$ and $c > 0$, then $a \times c > b \times c$.

If $a > b$ and $c < 0$, then $a \times c < b \times c$.

If $a > b$ and $c > 0$, then $a \div c > b \div c$.

If $a > b$ and $c < 0$, then $a \div c < b \div c$.

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STANDARDS FOR

Literacy in History/Social Studies, Science, and Technical Subjects

6–12

College and Career Readiness Anchor Standards for Reading

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Key Ideas and Details

1. Read closely to determine what the text says explicitly and to make logical inferences from it; cite specific textual evidence when writing or speaking to support conclusions drawn from the text.
2. Determine central ideas or themes of a text and analyze their development; summarize the key supporting details and ideas.
3. Analyze how and why individuals, events, or ideas develop and interact over the course of a text.

Craft and Structure

4. Interpret words and phrases as they are used in a text, including determining technical, connotative, and figurative meanings, and analyze how specific word choices shape meaning or tone.
5. Analyze the structure of texts, including how specific sentences, paragraphs, and larger portions of the text (e.g., a section, chapter, scene, or stanza) relate to each other and the whole.
6. Assess how point of view or purpose shapes the content and style of a text.

Integration of Knowledge and Ideas

7. Integrate and evaluate content presented in diverse formats and media, including visually and quantitatively, as well as in words.*
8. Delineate and evaluate the argument and specific claims in a text, including the validity of the reasoning as well as the relevance and sufficiency of the evidence.
9. Analyze how two or more texts address similar themes or topics in order to build knowledge or to compare the approaches the authors take.

Range of Reading and Level of Text Complexity

10. Read and comprehend complex literary and informational texts independently and proficiently.

*Please see "Research to Build and Present Knowledge" in Writing for additional standards relevant to gathering, assessing, and applying information from print and digital sources.

Note on range and content of student reading

Reading is critical to building knowledge in history/social studies as well as in science and technical subjects. College and career ready reading in these fields requires an appreciation of the norms and conventions of each discipline, such as the kinds of evidence used in history and science; an understanding of domain-specific words and phrases; an attention to precise details; and the capacity to evaluate intricate arguments, synthesize complex information, and follow detailed descriptions of events and concepts. In history/social studies, for example, students need to be able to analyze, evaluate, and differentiate primary and secondary sources. When reading scientific and technical texts, students need to be able to gain knowledge from challenging texts that often make extensive use of elaborate diagrams and data to convey information and illustrate concepts. Students must be able to read complex informational texts in these fields with independence and confidence because the vast majority of reading in college and workforce training programs will be sophisticated nonfiction. It is important to note that these Reading standards are meant to complement the specific content demands of the disciplines, not replace them.

Reading Standards for Literacy in History/Social Studies 6–12

RH

The standards below begin at grade 6; standards for K–5 reading in history/social studies, science, and technical subjects are integrated into the K–5 Reading standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of primary and secondary sources.	1. Cite specific textual evidence to support analysis of primary and secondary sources, attending to such features as the date and origin of the information.	1. Cite specific textual evidence to support analysis of primary and secondary sources, connecting insights gained from specific details to an understanding of the text as a whole.
2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of the source distinct from prior knowledge or opinions.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary of how key events or ideas develop over the course of the text.	2. Determine the central ideas or information of a primary or secondary source; provide an accurate summary that makes clear the relationships among the key details and ideas.
3. Identify key steps in a text's description of a process related to history/social studies (e.g., how a bill becomes law, how interest rates are raised or lowered).	3. Analyze in detail a series of events described in a text; determine whether earlier events caused later ones or simply preceded them.	3. Evaluate various explanations for actions or events and determine which explanation best accords with textual evidence, acknowledging where the text leaves matters uncertain.
Craft and Structure		
4. Determine the meaning of words and phrases as they are used in a text, including vocabulary specific to domains related to history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including vocabulary describing political, social, or economic aspects of history/social studies.	4. Determine the meaning of words and phrases as they are used in a text, including analyzing how an author uses and refines the meaning of a key term over the course of a text (e.g., how Madison defines *faction* in *Federalist* No. 10).
5. Describe how a text presents information (e.g., sequentially, comparatively, causally).	5. Analyze how a text uses structure to emphasize key points or advance an explanation or analysis.	5. Analyze in detail how a complex primary source is structured, including how key sentences, paragraphs, and larger portions of the text contribute to the whole.
6. Identify aspects of a text that reveal an author's point of view or purpose (e.g., loaded language, inclusion or avoidance of particular facts).	6. Compare the point of view of two or more authors for how they treat the same or similar topics, including which details they include and emphasize in their respective accounts.	6. Evaluate authors' differing points of view on the same historical event or issue by assessing the authors' claims, reasoning, and evidence.
Integration of Knowledge and Ideas		
7. Integrate visual information (e.g., in charts, graphs, photographs, videos, or maps) with other information in print and digital texts.	7. Integrate quantitative or technical analysis (e.g., charts, research data) with qualitative analysis in print or digital text.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, as well as in words) in order to address a question or solve a problem.
8. Distinguish among fact, opinion, and reasoned judgment in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claims.	8. Evaluate an author's premises, claims, and evidence by corroborating or challenging them with other information.
9. Analyze the relationship between a primary and secondary source on the same topic.	9. Compare and contrast treatments of the same topic in several primary and secondary sources.	9. Integrate information from diverse sources, both primary and secondary, into a coherent understanding of an idea or event, noting discrepancies among sources.
Range of Reading and Level of Text Complexity		
10. By the end of grade 8, read and comprehend history/social studies texts in the grades 6–8 text complexity band independently and proficiently.	10. By the end of grade 10, read and comprehend history/social studies texts in the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 12, read and comprehend history/social studies texts in the grades 11–CCR text complexity band independently and proficiently.

Reading Standards for Literacy in Science and Technical Subjects 6–12

RST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Key Ideas and Details		
1. Cite specific textual evidence to support analysis of science and technical texts.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to the precise details of explanations or descriptions.	1. Cite specific textual evidence to support analysis of science and technical texts, attending to important distinctions the author makes and to any gaps or inconsistencies in the account.
2. Determine the central ideas or conclusions of a text; provide an accurate summary of the text distinct from prior knowledge or opinions.	2. Determine the central ideas or conclusions of a text; trace the text's explanation or depiction of a complex process, phenomenon, or concept; provide an accurate summary of the text.	2. Determine the central ideas or conclusions of a text; summarize complex concepts, processes, or information presented in a text by paraphrasing them in simpler but still accurate terms.
3. Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks.	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.	3. Follow precisely a complex multistep procedure when carrying out experiments, taking measurements, or performing technical tasks; analyze the specific results based on explanations in the text.
Craft and Structure		
4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 6–8 texts and topics.*	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 9–10 texts and topics.*	4. Determine the meaning of symbols, key terms, and other domain-specific words and phrases as they are used in a specific scientific or technical context relevant to *grades 11–12 texts and topics.*
5. Analyze the structure an author uses to organize a text, including how the major sections contribute to the whole and to an understanding of the topic.	5. Analyze the structure of the relationships among concepts in a text, including relationships among key terms (e.g., *force, friction, reaction force, energy*).	5. Analyze how the text structures information or ideas into categories or hierarchies, demonstrating understanding of the information or ideas.
6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, defining the question the author seeks to address.	6. Analyze the author's purpose in providing an explanation, describing a procedure, or discussing an experiment in a text, identifying important issues that remain unresolved.
Integration of Knowledge and Ideas		
7. Integrate quantitative or technical information expressed in words in a text with a version of that information expressed visually (e.g., in a flowchart, diagram, model, graph, or table).	7. Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	7. Integrate and evaluate multiple sources of information presented in diverse formats and media (e.g., quantitative data, video, multimedia) in order to address a question or solve a problem.
8. Distinguish among facts, reasoned judgment based on research findings, and speculation in a text.	8. Assess the extent to which the reasoning and evidence in a text support the author's claim or a recommendation for solving a scientific or technical problem.	8. Evaluate the hypotheses, data, analysis, and conclusions in a science or technical text, verifying the data when possible and corroborating or challenging conclusions with other sources of information.
9. Compare and contrast the information gained from experiments, simulations, video, or multimedia sources with that gained from reading a text on the same topic.	9. Compare and contrast findings presented in a text to those from other sources (including their own experiments), noting when the findings support or contradict previous explanations or accounts.	9. Synthesize information from a range of sources (e.g., texts, experiments, simulations) into a coherent understanding of a process, phenomenon, or concept, resolving conflicting information when possible.
Range of Reading and Level of Text Complexity		
10. By the end of grade 8, read and comprehend science/technical texts in the grades 6–8 text complexity band independently and proficiently.	10. By the end of grade 10, read and comprehend science/technical texts in the grades 9–10 text complexity band independently and proficiently.	10. By the end of grade 12, read and comprehend science/technical texts in the grades 11–CCR text complexity band independently and proficiently.

College and Career Readiness Anchor Standards for Writing

The grades 6–12 standards on the following pages define what students should understand and be able to do by the end of each grade span. They correspond to the College and Career Readiness (CCR) anchor standards below by number. The CCR and grade-specific standards are necessary complements—the former providing broad standards, the latter providing additional specificity—that together define the skills and understandings that all students must demonstrate.

Text Types and Purposes*

1. Write arguments to support claims in an analysis of substantive topics or texts using valid reasoning and relevant and sufficient evidence.
2. Write informative/explanatory texts to examine and convey complex ideas and information clearly and accurately through the effective selection, organization, and analysis of content.
3. Write narratives to develop real or imagined experiences or events using effective technique, well-chosen details and well-structured event sequences.

Production and Distribution of Writing

4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach.
6. Use technology, including the Internet, to produce and publish writing and to interact and collaborate with others.

Research to Build and Present Knowledge

7. Conduct short as well as more sustained research projects based on focused questions, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, assess the credibility and accuracy of each source, and integrate the information while avoiding plagiarism.
9. Draw evidence from literary or informational texts to support analysis, reflection, and research.

Range of Writing

10. Write routinely over extended time frames (time for research, reflection, and revision) and shorter time frames (a single sitting or a day or two) for a range of tasks, purposes, and audiences.

*These broad types of writing include many subgenres. See Appendix A for definitions of key writing types.

Note on range and content of student writing

For students, writing is a key means of asserting and defending claims, showing what they know about a subject, and conveying what they have experienced, imagined, thought, and felt. To be college and career ready writers, students must take task, purpose, and audience into careful consideration, choosing words, information, structures, and formats deliberately. They need to be able to use technology strategically when creating, refining, and collaborating on writing. They have to become adept at gathering information, evaluating sources, and citing material accurately, reporting findings from their research and analysis of sources in a clear and cogent manner. They must have the flexibility, concentration, and fluency to produce high-quality first-draft text under a tight deadline and the capacity to revisit and make improvements to a piece of writing over multiple drafts when circumstances encourage or require it. To meet these goals, students must devote significant time and effort to writing, producing numerous pieces over short and long time frames throughout the year.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

The standards below begin at grade 6; standards for K–5 writing in history/social studies, science, and technical subjects are integrated into the K–5 Writing standards. The CCR anchor standards and high school standards in literacy work in tandem to define college and career readiness expectations—the former providing broad standards, the latter providing additional specificity.

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes		
1. Write arguments focused on *discipline-specific content.* a. Introduce claim(s) about a topic or issue, acknowledge and distinguish the claim(s) from alternate or opposing claims, and organize the reasons and evidence logically. b. Support claim(s) with logical reasoning and relevant, accurate data and evidence that demonstrate an understanding of the topic or text, using credible sources. c. Use words, phrases, and clauses to create cohesion and clarify the relationships among claim(s), counterclaims, reasons, and evidence. d. Establish and maintain a formal style. e. Provide a concluding statement or section that follows from and supports the argument presented.	1. Write arguments focused on *discipline-specific content.* a. Introduce precise claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that establishes clear relationships among the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly, supplying data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form and in a manner that anticipates the audience's knowledge level and concerns. c. Use words, phrases, and clauses to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.	1. Write arguments focused on *discipline-specific content.* a. Introduce precise, knowledgeable claim(s), establish the significance of the claim(s), distinguish the claim(s) from alternate or opposing claims, and create an organization that logically sequences the claim(s), counterclaims, reasons, and evidence. b. Develop claim(s) and counterclaims fairly and thoroughly, supplying the most relevant data and evidence for each while pointing out the strengths and limitations of both claim(s) and counterclaims in a discipline-appropriate form that anticipates the audience's knowledge level, concerns, values, and possible biases. c. Use words, phrases, and clauses as well as varied syntax to link the major sections of the text, create cohesion, and clarify the relationships between claim(s) and reasons, between reasons and evidence, and between claim(s) and counterclaims. d. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. e. Provide a concluding statement or section that follows from or supports the argument presented.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Text Types and Purposes (continued)		
2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic clearly, previewing what is to follow; organize ideas, concepts, and information into broader categories as appropriate to achieving purpose; include formatting (e.g., headings), graphics (e.g., charts, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with relevant, well-chosen facts, definitions, concrete details, quotations, or other information and examples. c. Use appropriate and varied transitions to create cohesion and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to inform about or explain the topic. e. Establish and maintain a formal style and objective tone. f. Provide a concluding statement or section that follows from and supports the information or explanation presented.	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize ideas, concepts, and information to make important connections and distinctions; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic with well-chosen, relevant, and sufficient facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among ideas and concepts. d. Use precise language and domain-specific vocabulary to manage the complexity of the topic and convey a style appropriate to the discipline and context as well as to the expertise of likely readers. e. Establish and maintain a formal style and objective tone while attending to the norms and conventions of the discipline in which they are writing. f. Provide a concluding statement or section that follows from and supports the information or explanation presented (e.g., articulating implications or the significance of the topic).	2. Write informative/explanatory texts, including the narration of historical events, scientific procedures/ experiments, or technical processes. a. Introduce a topic and organize complex ideas, concepts, and information so that each new element builds on that which precedes it to create a unified whole; include formatting (e.g., headings), graphics (e.g., figures, tables), and multimedia when useful to aiding comprehension. b. Develop the topic thoroughly by selecting the most significant and relevant facts, extended definitions, concrete details, quotations, or other information and examples appropriate to the audience's knowledge of the topic. c. Use varied transitions and sentence structures to link the major sections of the text, create cohesion, and clarify the relationships among complex ideas and concepts. d. Use precise language, domain-specific vocabulary and techniques such as metaphor, simile, and analogy to manage the complexity of the topic; convey a knowledgeable stance in a style that responds to the discipline and context as well as to the expertise of likely readers. e. Provide a concluding statement or section that follows from and supports the information or explanation provided (e.g., articulating implications or the significance of the topic).
3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)	3. (See note; not applicable as a separate requirement)

Note: Students' narrative skills continue to grow in these grades. The Standards require that students be able to incorporate narrative elements effectively into arguments and informative/explanatory texts. In history/social studies, students must be able to incorporate narrative accounts into their analyses of individuals or events of historical import. In science and technical subjects, students must be able to write precise enough descriptions of the step-by-step procedures they use in their investigations or technical work that others can replicate them and (possibly) reach the same results.

Writing Standards for Literacy in History/Social Studies, Science, and Technical Subjects 6–12

WHST

Grades 6–8 students:	Grades 9–10 students:	Grades 11–12 students:
Production and Distribution of Writing		
4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.	4. Produce clear and coherent writing in which the development, organization, and style are appropriate to task, purpose, and audience.
5. With some guidance and support from peers and adults, develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on how well purpose and audience have been addressed.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.	5. Develop and strengthen writing as needed by planning, revising, editing, rewriting, or trying a new approach, focusing on addressing what is most significant for a specific purpose and audience.
6. Use technology, including the Internet, to produce and publish writing and present the relationships between information and ideas clearly and efficiently.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products, taking advantage of technology's capacity to link to other information and to display information flexibly and dynamically.	6. Use technology, including the Internet, to produce, publish, and update individual or shared writing products in response to ongoing feedback, including new arguments or information.
Research to Build and Present Knowledge		
7. Conduct short research projects to answer a question (including a self-generated question), drawing on several sources and generating additional related, focused questions that allow for multiple avenues of exploration.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.	7. Conduct short as well as more sustained research projects to answer a question (including a self-generated question) or solve a problem; narrow or broaden the inquiry when appropriate; synthesize multiple sources on the subject, demonstrating understanding of the subject under investigation.
8. Gather relevant information from multiple print and digital sources, using search terms effectively; assess the credibility and accuracy of each source; and quote or paraphrase the data and conclusions of others while avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the usefulness of each source in answering the research question; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and following a standard format for citation.	8. Gather relevant information from multiple authoritative print and digital sources, using advanced searches effectively; assess the strengths and limitations of each source in terms of the specific task, purpose, and audience; integrate information into the text selectively to maintain the flow of ideas, avoiding plagiarism and overreliance on any one source and following a standard format for citation.
9. Draw evidence from informational texts to support analysis reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.	9. Draw evidence from informational texts to support analysis, reflection, and research.
Range of Writing		
10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.	10. Write routinely over extended time frames (time for reflection and revision) and shorter time frames (a single sitting or a day or two) for a range of discipline-specific tasks, purposes, and audiences.